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Public Debt and Growth:

An Assessment of Key Findings on Causality and Thresholds*

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Abstract

We provide a comprehensive assessment of the relationship between public debt and GDP growth in advanced economies. We use the timing of changes in public debt and growth to account for endogeneity and find no robust or sizable negative relationship in the full sample extending as far back as late 19th century. Semi-parametric estimates show no threshold effects. The relationship is essentially zero after 1970. We reconcile our results with four recent, influential papers that found a substantial negative relationship, especially when public debt exceeds 90 percent of GDP. These earlier results derive mostly from peculiar parametric specifications of nonlinearities or amplified influence of outliers in small samples.

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1 Introduction

The Great Recession and its aftermath have generated public debate about the effect of public debt on growth. Advocates of austerity argue that high public debt is a drag on economic growth. But causality may run largely in the other direction: slow economic growth reduces tax revenue and increases public spending, increasing fiscal deficits. Efforts to reduce debt by increasing taxes and decreasing spending during a downturn can aggravate macroeconomic problems in economies constrained by insufficient aggregate demand.

Important and influential empirical contributions include Reinhart and Rogoff (2010), Cecchetti et al. (2011), Checherita-Westphal and Rother (2012), and Woo and Kumar (2015). Based on overlapping samples of developed countries, these four papers make three key arguments. First, high public debt is associated with lower real GDP growth. Second, the relationship is the causal effect of public debt on growth. Third, above the important threshold of 90 percent in the public debt-to-GDP ratio, growth drops substantially. Recent papers have raised questions about the robustness and the causal content of the public debt-growth relationship. Herndon et al. (2014) identifies flaws in the Reinhart and Rogoff (2010) analysis of the bivariate relationship between public debt and growth, and Panizza and Presbitero (2014) instruments the public debt-to-GDP ratio with the valuation effect brought about by the interaction of foreign-currency debt with changes in the nominal exchange rate and finds that public debt does not reduce growth.

Using the superset of the developed countries in these studies from the late 19th century through 2011, we assess the relationship between public debt and growth, evidence on causality, and whether there are thresholds or nonlinearities. We first provide evidence on the timing of growth and public debt to assess causality and find that public debt is as closely (and sometimes more closely) associated with past growth as with future growth. We use an ensemble of standard tools to account for bias arising from the endogeneity of public debt: examining future (rather than contemporary) growth, instrumenting current public-debt-to-GDP ratio with its lag, and controlling for lagged GDP growth and other covariates. We formally establish that conditions under which each tool mitigates the bias due to reverse causality. The ensemble of these approaches generate broadly similar estimates, providing a robust assessment of the causal relationship.

While the contemporaneous bivariate estimate for the full sample suggests that a 100 percentage point increase in public debt is associated with a 1.5 percentage point reduction in real per capita annual GDP growth, moving from contemporary to future growth as the outcome diminishes the reduction to 1.1 percentage point. OLS and IV specifications that control for past growth, with or without country fixed effects and with or without control variables frequently used in growth equations, reduce the original estimate and produce statistically significant estimates of around 0.7 percentage points. When the sample is limited to years since 1970—which is the time period considered in most of the recent studies—there is no evidence of a negative association between public debt and future growth in any specification. Inclusion of a large set of covariates does not alter these conclusions.

We assess nonlinearities and threshold effects by showing the data with non-parametric and semi-parametric plots and minimal assumptions about functional form. The plots establish that the relationship between public debt and growth is essentially flat for public debt/GDP exceeding 50 percent when we examine future growth or control for past growth. Contemporaneous public debt is as (or more) negatively associated with growth in the preceding five years, as it is with 5-year forward growth. We conclude that causality more likely runs from GDP growth to public debt than vice versa. There are no systematic thresholds at elevated levels of public debt in the data; 5-year forward growth rates are no lower when public debt crosses 90 percent of GDP.

In addition to providing estimates based on a superset of the data with a flexible modeling strategy, we examine the four papers in detail to reconcile the results. Earlier results are sensitive to model and sample selection. In particular, we find that parametric models of turning points (such as a quadratic specification) can produce misleading inference. In studies with smaller samples, a single influential country (e.g., Ireland) produces a likely spurious inverse U-shaped relationship between public debt and growth. In other cases, the negative estimate requires particular choices of controls and specifications and disappears with minor variations.

Section 2 lays out the identification problem, explains the specifications, and discusses the data. Section 3 presents the results from the linear specifications, non- and semi-parametric models, and replicationsof the samples and specifications used in four key papers. Section 4 summarizes key findings and concludes.

2 Empirical Strategy

2.1 Endogeneity of debt and the identification problem

The identification challenge is that public debt is endogenous as an outcome of growth. A posited causal negative relationship from public debt to growth depends on crowding-out mechanisms. Public borrowing may raise interest rates, which can crowd out private investment or, via exchange rate appreciation, reduce net exports. Inflation associated with government debt may act as a drag on growth, although neither mechanism, from public debt to inflation nor from inflation to growth, is self-evident. High public debt also creates vulnerability to interest rates when public debt is rolled over.

However, pubic debt is endogenous, and may reflect both resverse causality and omitted confounders that could influence both growth and debt. For example, lower economic growth reduces tax collections and increases public spending, e.g., on unemployment insurance; fiscal deficits increase through the operation of automatic stabilizers.

We formalize the reverse causality problem with the following two equations. The first is growth, which depends on public debt,

$$g_{it} = D_{it}b_1 + u_{it} \tag{1}$$

in which g_{it} represents the growth rate of GDP for country i in year t, D_{it} represents the stock of public debt, b_1 is a parameter, and u_{it} represents the error term and includes all other controls or contributors to GDP growth. The second equation describes the evolution of public debt,

$$D_{it} = g_{it}a_1 + v_{it} \tag{2}$$

which introduces the parameter a_1 and error term v_{it}. If b_1 is reasonably small and $a_1 < 0$, then the OLS bias in $\hat{b}_1$ is negative, i.e., public debt is estimated to be worse for GDP growth than it actually is.[1]

To assess the endoeneity problem we begin by considering the sequencing of public debt and growth: which comes first? We test the importance of sequencing by examining leads and lags of GDP growth in relation to public debt. To mitigate some of the bias from sequencing, our preferred outcome $\bar{g}_{i,t+1,t+k}$ averages the growth rate of GDP over the succeeding k years.[2]

However, a correlation between contemporaneous public debt and future GDP growth may indicate complex lags rather than causality. To account for such dynamics, we also

[1] There is also a mechanical negative relationship between growth and public debt/GDP in that lower GDP growth is mechanically associated with a lower denominator in public debt/GDP; so lower growth and higher public debt/GDP can be associated through the common term, without any actual economic impact of public debt on growth.For example, if GDP is measured with error, then a spurious negative relationship will obtain between growth, $\log(GDP_t) - \log(GDP_{t-1})$, and the public debt-to-GDP ratio, Public Debt_t/GDP_t. Measurement error in GDP_t will induce a spuriously negative regression coefficient. A positive error in the measurement of current GDP_t both increases measured growth and decreases the measured ratio of public debt to GDP. Thus, measurement error will bias the correlation in a negative direction.

[2] Of the four papers we re-examine, only Reinhart and Rogoff (2010) focuses exclusively on the contemporaneous relationship between public debt and growth. The other papers examine the relationship between contemporary public debt and five-year forward GDP growth.

employ specifications which control for lagged growth:

$$\bar{g}_{i,t+1,t+k} = \beta \cdot d_{i,t} + \rho \cdot g_{i,t-1} + \gamma_t + \varepsilon_{i,t} \tag{3}$$

The control for lagged growth thus accounts directly for past downturns, which may be simultaneously responsible for high debt (because of the fiscal deficits accumulate) and continued low growth (because recessions linger).

We also estimate models with country fixed effects which account for time-invariant sources of heterogeneity across countries, a specification common in the literature.

$$\bar{g}_{i,t+1,t+k} = \beta \cdot d_{i,t} + \mu_i + \gamma_t + \varepsilon_{i,t} \tag{4}$$

However, the fixed effects specification does not account for time-varying heterogeneity and can be particularly susceptible to endogeneity bias. For this reason, as an additional approach to account for the endogeneity of public debt, we instrument current public debt with 5-year lagged public debt. The IV specification limits identification to public debt in places where public debt is persistent, i.e., where current public debt is explained by a history of public indebtedness rather than by macroeconomic shocks. We estimate both the fixed-effect and lagged-growth specifications by IV.

In Appendix A, we demonstrate the bias and formally derive the conditions under which the approaches mitigate the bias in the bivariate estimate of contemporaneous growth on public debt. In particular, we find that under plausible conditions, use of forward GDP growth, controls for lagged growth, and use of lagged debt instrument all reduce the magnitude of the bias in the estimated causal effect of public debt on growth.

Finally, we use our identification strategy to assess the proposed crowding-out mechanisms by which public debt may negatively affect economic growth. In lieu of real per capita GDP growth, we apply the model to real total factor productivity (TFP) growth. TFP growth describes economic growth after accounting for factor accumulation. If public debt primarily interferes with capital accumulation, then TFP growth should be less affected than GDP growth, which reflects both capital accumulation and TFP growth.

2.2 Non-Linearities and Thresholds

All four papers surveyed here emphasize the importance of nonlinear effects and a threshold above which the relationship between public debt and growth becomes more negative. Here we review the particular approaches taken in those studies and how they differ from what we do in this paper.

Reinhart and Rogoff (2010), which examines real GDP growth stratified by discrete categories for ranges of the public debt-to-GDP ratio, write, "it is evident that there is no obvious link between debt and growth until public debt reaches a threshold of 90 percent. The observations with debt to GDP over 90 percent have median growth roughly 1 percent lower than the lower debt burden groups and mean levels of growth almost 4 percent lower" (p. 575). As shown in Herndon et al. (2014), the apparent nonlinearity was not a robust finding and was driven by a number of peculiar choices and errors.

Checherita-Westphal and Rother (2012), Cecchetti et al. (2011), and Woo and Kumar (2015) all use parametric methods to identify nonlinearities in the relationship between public debt and GDP growth. Checherita-Westphal and Rother (2012) uses a quadratic specification without establishing that it is appropriate – and reports the turning point: "The debt-to-GDP turning point of this concave relationship (inverted U-shape) is roughly between 90 and 100% on average for the sample, across all models. This means that, on average for the 12 euro area countries, government debt-to-GDP ratios above this threshold would have a negative effect on economic growth" (p. 1398). An implication of the quadratic specification is that the relationship between debt and growth is symmetric around the turning point. If debt really has a negative causal effect above the threshold, a quadratic specification imposes a symmetric positive effect below it.

Woo and Kumar (2015) implements a three-segment linear spline with the segments comprising public debt-to-GDP ratios of 0–30 percent ("low"), 30–60 percent ("medium"), and 60–90 percent ("high"). The breakpoints coincide with those identified by Reinhart and Rogoff (2010) as marking high and low levels of public debt/GDP. Woo and Kumar (2015) finds that the magnitude of the negative relationship between public debt and GDP growth increases at higher levels of public debt.[3]

The Woo and Kumar (2015) specification permits the slopes of the linear segments to vary, but the segments are not continuous and are constrained to a single intercept on the vertical axis, i.e., at a public debt-to-GDP ratio of zero, and are hence not readily interpreted. A schematic of the spline specification issue, which also affects the Cecchetti et al. (2011) threshold analysis, is illustrated in Appendix Figure D.1.[4]

Cecchetti et al. (2011) uses a semi-parametric break-point method for identifying the threshold. The study implements a two-segment linear spline with an unknown threshold, with the best-fit threshold identified on the basis of minimizing the sum of squared residuals.[5]

[3] Woo and Kumar (2015) report, "The coefficients of low initial debt (i.e., initial debt∗Dum_30) are insignificant and even change sign [with some estimation methods]. In the OLS, the coefficient of medium level of debt (initial debt∗Dum_30–90) is significant at 5 percent, and its estimated coefficient is −0.025. But they are all insignificant [using the other estimation methods]. By contrast, the coefficients of high debt (initial debt∗Dum_90) are negative and significant under [all estimation estimation methods except country fixed effects]." These results are based on a sample of advanced and developing economics; results for a spline specification for the advanced economies are not reported.

[4] Standard alternatives include a knotted spline that forces continuity (but not differentiability) by knotting the linear segments at the breakpoints or an unconstrained spline that permits both discontinuous jumps at the breakpoints, i.e., threshold effects, and different slopes in the segments. Woo and Kumar (2015) includes only the interacted terms but not the main effects of the indicators for the public debt/GDP segments.

[5] Cecchetti et al. (2011) reports that "96 percent of GDP is the point estimate of the threshold level. At the 1 percent confidence level, the threshold level lies between 92 percent and 99 percent of GDP — that is, the level at which we estimate that public debt starts to be harmful to growth may be as low as 92 percent of GDP and as high as 99 percent (using 5 percent or 10 percent confidence levels would not change the interval much)" (p. 167). Cecchetti et al. (2011) implement the threshold in their Equation (2):

$$\bar{g}_{i,t+1,t+k} = -\phi y_{i,t} + \beta X_{i,t} + \lambda_{-} \cdot d_{i,t}\mathbf{I}(d_{i,t} < \tau) + \lambda_{+} \cdot d_{i,t}\mathbf{I}(d_{i,t} \geq \tau) + \mu_i + \gamma_t + \varepsilon_{i,t,t+k},$$

where $\mathbf{I}(.)$ is the indicator function, τ is a threshold, $\bar{g}_{i,t+1,t+k}$ is average growth rate of GDP over the succeeding k years, y is level of GDP, d is public debt/GDP, X represents a set of controls, and $\lambda_{-/+}$ is the responsiveness of GDP growth to public debt/GDP below/above the threshold.

Thus neither Woo and Kumar (2015) nor Cecchetti et al. (2011) knots the spline segments at the threshold or implements a fully unconstrained spline that permits both an unconstrained discontinuity and a change in slope at the threshold. The resulting specification has discontinuity at the threshold but the coefficients on the slope of the segments are constrained because both segments must share a single common y-intercept.

Although these formal methods identify turning points from more positive to more negative slopes, they do not distinguish among a wide variety of concave shapes, from cliffs to inverted-U's. Even if the threshold findings in Cecchetti et al. (2011) Checherita-Westphal and Rother (2012), and Woo and Kumar (2015) are accurate and causal, they are consistent with an inconsequential relationship between public debt and growth in the policy-relevant region.

In contrast, we employ non-parametric and semi-parametric methods to examine nonlinearities, which allows us to show the relationships transparently without imposing parametric assumptions. First, we use a bivariate lowess-smoothed regression to summarize the potentially nonlinear relationship between contemporary public debt and lagged, contemporary, and future GDP growth at alternative levels of public debt without assuming functional forms or thresholds. We then use the partial linear model (DiNardo and Tobias, 2001) that implements our preferred regression specifications with linearly controls for past GDP growth and time effects while permitting a fully flexible relationship between public debt and future GDP growth:[6]

$$\bar{g}_{i,t+1,t+k} = f(d_{i,t}) + \rho \cdot g_{i,t-1} + \gamma_t + \varepsilon_{i,t}$$

We also estimate analogous PLM regressions controlling for country fixed effects instead of lagged growth.[7]

[6] We implement the partial linear model (PLM) with `plreg` in STATA.

[7] Countries may differ in the historical levels of public debt and growth and in the country-specific relationship between public debt and growth. We do not assess such heterogeneity in the non-linear effect

2.3 Data on Debt and Growth

In this paper, we use data from International Monetary Fund to construct a dataset of 22 developed countries that subsumes the sample of developed countries used in Reinhart and Rogoff (2010), Cecchetti et al. (2011), Checherita-Westphal and Rother (2012), and Woo and Kumar (2015).[8] In no case were the data archived for replication by the journals or authors; we construct sub-samples of our data, RR1955 (20 countries), CMZ (18 countries), CWR (12 countries) and WK (18 countries), that replicate the country and year selections in these papers. The details of each subsample are given in Table 1. We accept the data definitions and values as presented by the authors.[9]

Our specifications require 5 years of leads and lags. Startpoints are limited by country-specific data availability, and the endpoint is 2011. To clarify, since our preferred outcome is the five-year forward averaged growth, while our estimates use growth information through 2016, they are based on public debt information through 2011. For the replication components, we employ the years analyzed in each paper. For RR1955 we limit the sample to 1955 to 2003; our sample for RR1955 thus excludes the early-postwar observations that Herndon et al. (2014) demonstrated were highly influential in Reinhart and Rogoff (2010). For CMZ we keep observations from 1985 to 2003. For CWR and WK we keep observations from 1975 to 2003.[10]

in this paper, instead estimating the average relationship between debt and growth across countries. See Panizza and Presbitero (2013) for a discussion of this issue, which is a potential direction for future research.

[8]We draw the key variables from Mauro et al. (2013). The Mauro et al. (2013) closely resemble the Abbas et al. (2010), both of which are provided by the IMF. We use the Mauro data based on fewer missing observations for the years and countries in the analysis. Additional variables were drawn from Heston et al. (2011) and the World Development Indicators (World The World Bank, 2017).

[9]Requests for replication data to the corresponding authors for Checherita-Westphal and Rother (2012) and Woo and Kumar (2015) did not receive a response. Access to the data for Reinhart and Rogoff (2010) is discussed in Herndon et al. (2014).

[10]We also examined how differences in sample years and sample countries, as opposed to differences in method, affect the results by constructing common samples between pairs of datasets and computing the main results for the common samples. These results, available from the authors, strengthen our substantive conclusions about the relationship between public debt and growth.

3 Results

3.1 Trends in GDP growth and public debt in advanced economies

The upper panel of Figure 1 shows the counts of countries (out of 22 in the full sample) for which the public debt-to-GDP ratio was available and exceeded 60 percent or 90 percent over the period 1880–2011. There are large swings in public debt around the World Wars. The period since World War II shows two notable patterns. From the the end of World War II through the late 1960s, the prevalence of high public debt decreased, in part because several countries finished reducing high public debt from World War II. By the middle 1970s, no country had public debt above 60 percent of GDP. Beginning in the late 1970s prevalence of higher public debt increases. By the middle 1990s, 13 of the 22 countries had public debt above 60 percent, and the number of countries with public debt above 90 percent of GDP hovered around 5. The prevalence of public debt in excess of either 60 percent or 90 percent of GDP increased sharply after the Great Recession began in 2007–2008.

The lower panel of Figure 1 shows the interquartile range of annual real per capita GDP growth using all available data for the 22 countries for 1880–2011. Focusing again on the period since World War II, the end of the golden age in the early 1970s is evident in the data. Even laggards of the 1960s had growth similar to the fastest growers in the 1980s and 1990s.

Growth slowed sharply in the 1970s when public debt was near its nadir. During and after the 1980s growth remained slow relative to the postwar boom and public debt grew. The postwar data suggest a negative relationship between growth and public debt. The time series plots broadly suggest that high growth accompanied debt decreases in the 1960s and that lower growth preceded the growth of public debt more recently, but it is hard to disentangle the direction of causality.

3.2 Regression Results using Linear Models

Table 2 shows OLS and IV regression results for the linear relationship between public debt and current growth and between public debt and future growth using the full sample for the maximum available years. All models specify a linear relationship between real per capita GDP growth and public debt/GDP. We report the change in average annual growth rate in percentage points associated with a 1 percentage-point increase in the public debt-to-GDP ratio.

Our coefficient of -0.015 in column 1 indicates that a 100 percentage-point increase in the public debt-to-GDP ratio would be associated with 1.5 percentage point lower real annual GDP growth. The implied effect on growth of a 100 percentage-point increase in the public debt-to-GDP ratio — for example, the change in Japan from its low-debt postwar boom to the 2000s when its public debt exceeded 100 percent of GDP — implies 1.5 percentage points per year lower real annual GDP growth, a substantial implied reduction in growth associated with higher public debt. When no other controls are used, the results using the full sample and contemporaneous public debt and GDP growth data—the regression specification implied by Reinhart and Rogoff (2010) analysis,—find a statistically significant negative relationship between public debt/GDP and current growth. The introduction of year dummy variables, with results reported in column 2 of row 1, substantially reduces the relationship from -0.015 to -0.010, but preserves statistical significance.

Of greater interest with respect to a causal relationship between public debt and growth are the results for future growth reported in row 2 of Table 2. We find a coefficient of -0.011, which falls to -0.008 with the addition of year dummies in column 2. When we add one lag of GDP growth as an explanatory variable, the estimated coefficient remains at -0.008 .

We then switch from the inclusion of lagged GDP growth as a control to the use of lagged public debt as an instrument for contemporaneous public debt. In the first stage results, reported in detail in Appendix Table D.7, the coefficient on once-lagged debt, the excluded exogenous instrument, is of the expected positive sign — and approximately 1 throughout, which raises the issue of non-stationarity of public debt — which indicates that past debt does partially explain present debt. The F-statistic for the excluded exogenous instrument is always substantially greater than 10. Overall we conclude that lagged debt is a meaningful predictor for current debt, and we argue that sequence makes a reasonable case for the exclusion restriction, i.e., that lagged debt is appropriately excluded from a regression of growth on current debt. In the instrumental variables result and when both methods of addressing endogeneity are applied, the coefficient on public debt remains stable at -0.007. The point estimate implies that 100 percentage points of additional public debt reduce growth by 0.7 percentage point.

Returning to row 1, in which current growth is the dependent variable, and examining the alternative methods of addressing endogeneity, the results are rather similar in magnitude to the results with forward growth as the dependent variable. In columns 3, 4, and 5 of row 1, the estimated coefficient ranges from -0.004 to -0.012. The result gives some confidence in the methods of using lagged GDP growth as a control and lagged public debt as an instrument in that the current growth specification, which is clearly contaminated by endogeneity, yields essentially the same results as the forward growth specification.

Overall we take row 2, column 5 of Table 2 as our preferred linear specification because it examines forward growth, accounts for inertia in business conditions by controlling for lagged growth, and uses the instrument of lagged public debt to identify with "chosen" public debt rather than endogenous public debt. In this specification the coefficient on public debt/GDP is -0.007 and is significant with a p-value just over 0.05.

The final two columns (columns 6 and 7) of Table 2 use country fixed effects, with public debt instrumented by its lag in column 7. We do not include the lag of growth in the fixed-effect models because of the requirement of strict exogeneity in the estimation of fixed-effect panel models (Wooldridge, 2010). The results diverge substantially between outcomes of current and future growth and the use of IV. The IV fixed-effect results in rows 1 and 2 of column 7 show minimal effects, estimates of 0.000 and −0.004, neither statistically significant.

Rows 3 and 4 review the baseline specification in row 2 with additional control variables that may explain growth and hence reduce the endogeneity problem with public debt, drawing from other papers in the literature. The covariates include: population, population growth, and the dependency ratio to represent demographic status and trends; GDP per capita, average years of secondary schooling (Barro and Lee, 2013), and the urban share of the population to capture the level of development; the real effective exchange rate and trade relative to GDP to capture international orientation; inflation in consumer prices, gross domestic savings as a percent of GDP, and general government final consumption expenditure as a percent of GDP as potential indicators of crowding out or slack; and the stock of mortgages relative to GDP (Jorda et al., 2017) as an indicator of financial-market development and credit-market activity. These variables are available for only a subset of the countries and years; so sample size is substantially reduced. In row 3 we present the baseline specifications, equivalent to row 2 but with the sample restricted to the observations for which all control variables were available. The similarity of results in row 2 and row 3 bolsters confidence that in row 4 we will see the effect of the additional control variables rather than a reduced sample. In row 4, the inclusion of additional controls does not substantially alter any of the relationships estimated in row 2. Since the key findings are unaffected from use of these covariates while the sample size is substantially diminished, we do not include them in the analysis that follows.

In Table 3, we examine the proposed crowding-out mechanism by which public debt may affect GDP growth. Row 1 replicates row 2 of Table 2 limited to the subsample for which TFP growth data were available, and results are very similar. Row 2 presents the same specifications with TFP growth rather than GDP growth as the dependent variable. The TFP results and the GDP results are very similar, which suggests that if there is a negative effect of public debt on economic growth, the pathway is not crowding out of capital accumulation.

3.3 Sensitivity analysis: Alternative Leads and Lags

Figure 2 probes the sensitivity of the baseline regression specifications to alternative lags and leads in the definition of the dependent variable, to the inclusion of alternative lags of the dependent variable as control variables, and to the sample period. The vertical axis reports the estimated coefficient on public debt in response to 11 alternative definitions of the window for the dependent variable, from 5 years lagged growth through current growth to 5 years forward growth, plotted on the horizontal axis. For cases with contemporaneous or forward growth as the dependent variable, we also examine 3 alternative controls for lagged growth: no control for lagged growth (indicated by green circles); 1-year lagged growth (orange triangles); and 5-year lagged growth (blue diamonds). Throughout, we use filled shapes to indicate statistical significance at the 5-percent level.

Figure 2 presents results for the full sample in the upper panel. The solid green circle in the middle of the upper panel shows the coefficient of −0.010 in the regression of contemporaneous growth on public debt/GDP, which corresponds to row 1, column 2 of Table 2. The introduction of 1-year lagged growth or 5-year lagged growth as a control variable in the contemporaneous regressions has essentially no effect on the estimate.

When we examine increasing windows of forward growth the coefficient on public debt/GDP remains stable in magnitude between−0.010 and −0.008, which corresponds to the difference between rows 1 and 2 of column 2 in Table 4.Controls for lagged growth leave the estimate stable as well. With past growth as the dependent variable, the coefficients remains stable between between−0.010 and −0.007 as well. In other words, the relationship between contemporaneous public debt and past growth is about as strong as it is with future growth. Overall, it seems unlikely that this pattern primarily reflects the effect of public debt on growth, and instead indicates an endogeneity in the relationship.

3.4 Results from Alternative Subsamples

Table 4 tests the sensitivity of the main results, i.e., future growth with the range of specifications, to use of the four paper-specific subsamples described in Table 1. The RR1955 results range from -0.003 in the specification with both lagged GDP growth as a control and lagged public debt as an instrument to -0.019 in the specification with fixed effects and no other controls; only this latter specification is significant with a p-value below 0.05.

The results for the other three samples are quite different. The coefficients in all specifications for the other three samples, CMZ, CWR, and WK, range from essentially zero to, in several cases, positive and significant. In columns (6) and (7), the CMZ sample shows a significant positive relationship between public debt and future growth. This result is especially surprising because Cecchetti et al. (2011) reports a significant negative coefficient for the linear model including fixed effects; we return to the CMZ results in more detail below. The CWR sample including country fixed effects and with or without IV also generates substantively and significantly large positive effects of public debt on GDP growth. In columns 3, 4, 5, 6, and 7 of row 4 of Table 6, the coefficient on public debt is effectively zero and never statistically significant, including in the specifications with endogeneity controls and with fixed effects.

The CMZ, CWR, and WK subsamples all have fewer countries and later starting years than RR 1955 or our full sample. Further exploration finds that the relationship between public debt and growth weakened after 1970, an important result because of its relevance for contemporary policy and because 3 of the 4 papers reviewed here focus on this period. With the same structure as the upper panel, the lower panel of Figure 2 tests the sensitivity to alternative leads and lags of the sample limited to 1970–2011. In the more recent data, the only statistically significant negative relationship between public debt and growth is for the contemporaneous relationship with no controls for lagged growth. The relationship between public debt and future growth at any lead is small and insignificant, falling by about half with a one-year lead and shrinking to essentially zero by five years out. Introducing a control for lagged growth with either contemporaneous or forward growth as the dependent variable yields estimates that are very close to zero. The implication of row 4 of Table 6 and the lower panel of Figure 2 is that the effect of public debt on GDP growth in the post-1970 period is close to zero.

Table 5 examines the public debt and growth with the full sample of countries for the full sample of years and for years since 1955, 1960, 1965, and 1970 to test whether the effect of public debt on GDP growth has remained stable over time. In columns 3, 4, and 5 of row 3, which all use current growth as the dependent variable with the three specifications proposed to address endogeneity, the coefficient on public debt ranges from -0.001 to -0.003. All three are precisely estimated. When country fixed effects are included in columns 6 and 7 of row 3, the estimated coefficient on public debt increases somewhat in magnitude from the estimates without fixed effects. In column 6 with fixed effects but no endogeneity controls, the magnitude is similar in size to the estimate on the post-1955 data. When lagged public debt is introduced as an instrument for contemporaneous public debt, the magnitude of the estimated coefficient falls by about half, to -0.007. To re-iterate, in the sample after 1970—which covers the samples in Cecchetti et al. (2011), Woo and Kumar (2015), and Checherita-Westphal and Rother (2012)— there is no evidence of a statistically significant or substantial negative relationship between forward growth and public debt in any of the OLS or IV specifications, and with or without controls for lagged growth.

The RR1955 data, which closely resemble the full sample, yield similar results as shown in Table 2. For the CMZ, CWR, and WK subsamples, the results in Table 4 never include significant negative estimates for the bivariate relationship in either the contemporaneous or forward data. We conclude that the negative relationships between public debt and GDP growth found in Cecchetti et al. (2011), Woo and Kumar (2015), and Checherita-Westphal and Rother (2012) likely represents artifacts of specification, sample inclusion, and functional form, which we explore in more detail below.

3.5 Nonlinearities and Thresholds

Although linear regression summarize the relationship between public debt and GDP growth, the literature emphasizes non-linearities and identifies an important threshold in the relationship between the public debt-to-GDP ratio and economic growth. Based on the stability of the linear results reported in Figure 2, five-year forward growth remains the key outcome indicator in the nonlinear analysis.We report bivariate results for the full sample, multivariate results for the full sample limited to 1970-2011, and multivariate results for the subsamples used in the four papers.

Figure 3 shows the bivariate lowess estimates of GDP growth versus public debt/GDP for the full sample. The left panel plots 5-year lagged GDP growth versus current year public debt/GDP, thereby capturing the reverse-causal relationship from growth to public debt. The middle panel of each triptych shows current GDP growth versus current-year public debt-to-GDP ratio. Strikingly, while the contemporaneous relationship is downward sloping across datasets, the slope is generally less pronounced than in the lagged-outcome panel. The fact that contemporary pubic debt is more strongly associated with past growth rather than current growth indicates reverse causality. The right panel shows future growth, expressed as the annualized average real GDP growth over the succeeding five years versus current-year public debt-to-GDP ratio. Between public debt-to-GDP of 30 percent and 150 percent, the policy relevant range for current debates, the lowess plots indicate a small to zero relationship between public debt and future growth in any of the datasets.

While the bivariate relationships are illustrative, they do not control for the same variables as do the linear regression results. Therefore, we next use a partial linear model to linearly control for these covariates while allowing a nonparametric relationship between public debt and growth. Results for the full sample and the full sample limited to 1970-2011 are shown in Figure 4. These figures show alternative specifications, one with a control for lagged growth and one with country fixed effects. Both panels show essentially no relationship between public debt and growth. To the extent that there is a relationship, it obtains between 0 and 30 percentage points of public debt-to-GDP.

Results for all of the datasets are shown in Figures D.2. We control for recent growth by including 1-lag of GDP growth in the partial linear model shown in Figure D.2, which also shows the actual scatterplot of data. (Results including country fixed effects are shown in the Appendix in Figure D.3.)

The lowess curves that control for lagged GDP growth are generally rather flat, indicating little relationship. One exception is CWR where the relationship appears as a shallow rotated-S shape. There is a turning point at roughly 90 percent, but the inverted-U shape at that location is shallow. Accounting for confidence intervals, the estimated function is indistinguishable from a horizontal line. Overall there is little relationship between public debt and future GDP growth once lagged GDP growth is taken into account.

Although essentially no relationship to GDP growth appears at medium to high levels of public debt, the relationship at low levels of public debt varies by dataset. In both the Full Sample and RR1955 subsample, most of the observed decline in GDP growth occurs, contrary to debt-threshold hypotheses, in the first 50 percentage points of public debt/GDP. There is no sign of a negative relationship in the Full Sample limited to the period after 1970. In the case of CMZ there is a sharp increase in growth as public debt ranges from 16 percent, the lowest value of public debt/GDP in the CMZ data, to 50 percent of GDP followed by a horizontal (non-)relationship. CWR and WK show increasing growth up to a public debt-to-GDP ratio of 100 percent and a flat relationship thereafter.

The inclusion of scatterplot data in Figure 5 elucidates some of the summary findings.

The full sample after 1970, the RR1955 data, the CMZ data, and the CWR data all show one or more especially fast-growing countries in the medium-high public debt range. In the Full Sample, RR1955, and CWR data, the case is Ireland in the early 1990s, while in the CMZ data, the medium-high-debt, high-growth case is Portugal in the middle 1980s. These potentially idiosyncratic country-years may be particularly important for identification of turning points in Checherita-Westphal and Rother (2012) and Cecchetti et al. (2011) and illustrate the effect of high-leverage cases.

To summarize, the relationship between public debt and forward growth appears modest across a variety of specifications. Taken together the semi-parametric plots including country and year fixed effect indicate small to no reductions in growth which largely disappear in more recent data. Importantly, we find no evidence across various specifications of any clear threshold at elevated levels of public debt, and this is true whether we control for heterogeneity using fixed effects or lagged outcomes. These findings stand in contrast to claims in the four key studies we review here. Therefore, in the next sections we scrutinize the findings of both linear effects and nonlinearities in these paper in greater detail.

3.6 Specific issues in replication

Since our findings using the same countries and time periods as the previdous papers often appear to be quite different than found by the authors, here we discuss some of the specific factors that can reconcile these differences.

3.6.1 Sensitivity to influence points in Checherita-Westphal and Rother (2012)

Checherita-Westphal and Rother (2012) report a threshold effect based on a quadratic specification to identify a turning point in the relationship between public debt and GDP growth. The analysis finds the peak of the quadratic to be at a public debt-to-GDP ratio between 90 and 100 percent. It is important to keep in mind what is meant by the "turning point of this concave relationship." Taken at face value, the implication is that public debt has a growth-enhancing effect up until the turning point. The results reported in the paper imply that a public debt-to-GDP ratio of 60 percent, the maximum public debt level permitted under the Maastricht agreement and subsequent European Stability and Growth pact, reduce growth by roughly one percentage point of annual GDP growth relative to maintaining public debt/GDP at the peak around public debt/GDP of 90 and 100 percent. Another feature of a quadratic is that around the peak the slope is necessarily close to zero, which means that changes in public debt are unlikely to have a large effect on growth.

In Figure D.4 the partial linear regression plot from the CWR data presents a rotated-S shape, with a trough of annual GDP growth of 2.03 percent at public debt/GDP of 35 percent and a peak of annual GDP growth of 2.73 percent at public debt/GDP of 90 percent. Although the shape is dramatic and the estimate at high public debt/GDP is reasonably precise (in terms of the width of the error bands), the net relationship — both the positive relationship between public debt/GDP of 35 and 90 percent and the negative relationship between public debt/GDP of 90 and 140 — does not amount to much. With relatively close values of annual GDP growth at the trough and peak ($2.73 - 2.03 = 0.7$ percentage points of GDP growth), there is simply not much room for a strong relationship between public debt and GDP growth. A horizontal line at annual real GDP growth 2.5 would miss some of the nonlinearity, but it would be hard to reject as a characterization of the relationship between public debt and growth in these data.

Visual inspection of the actual scatterplot of CWR data in the CWR panel of Figure D.4 indicates a highly visible and potentially influential set of points that trace out a high arc. Thus the actual scatterplot underlying the rotated S-shape of the fit to the CWR data indicates that there are one or two countries with high future growth associated with moderately high public debt. That is, the curve is being pulled up as public debt/GDP approaches 90 percent from below. As we noted above, these outlying cases are Portugal in the middle 1980s and Ireland in the early 1990s.

Focusing on the data points from Ireland, that country's peak growth occurred in the mid-1990s when public debt was between 70 and 80 percent of GDP. We estimate partial linear regressions plots with and without the inclusion of the Ireland in Appendix Figure D.6. Ireland proves to be fairly influential in generating the rotated-S shape in the relationship, a result that obtains with and without fixed effects. Indeed, without Ireland, the relationship between public debt and growth is essentially flat in the model that controls for lagged growth and modestly upward sloping in the model with fixed country effects.

Ireland has a particularly important effect in the CWR results because the CWR data are limited to twelve countries. The same data for Ireland exert less influence in our full sample and the other large datasets simply because they represent a smaller share of the data points. This points to the pitfalls of using a small sample, where inference can be driven by a single influential case.

The results also support the proposition that parametric trend-break tests can be misleading, and so it is important to examine the data directly as well. The nonparametric curve in the CWR sample does turn downward at public debt/GDP above 90 — a fact captured by the maximum of the quadratic in CWR's regression — but it turns downward from a set of very high growers among countries with moderate-to-high public debt. It is not that growth collapses at public debt/GDP of 90 percent but rather that there are a small number of high growth cases immediately below 90 percent public debt/GDP.

Overall, our re-analysis of the CWR sample suggests that there is little overall relationship between public debt and growth in their sample. There is an indication of a non-monotonic relationship between the variables, which suggests a threshold effect of debt on growth. However, this inference is driven almost entirely due to a combination of a parametric test along with a single influential country (Ireland) in a small sample. Either using more data, or excluding that influential case, suggests there is no such threshold effect.

3.6.2 Sensitivity to sample period in Reinhart and Rogoff (2010)

Sensitivity to years was a central issue in the interpretation of the initial Reinhart and Rogoff (2010) finding, with Herndon et al. (2014) noting the particular importance of the treatment of the immediate postwar. In web-published errata to the original paper, Reinhart and Rogoff (2013) bypassed the question of the immediate postwar by selecting 1955 as the starting date for the revised analysis and found a modest negative relationship, without non-linearity, in the contemporaneous relationship between public debt and growth. However, the 1955 starting point substantially predates the starting dates for the other three papers on public debt and growth in the postwar advanced economies 1). The full sample limited to years after 1970 finds that the negative relationship between public debt and GDP growth largely disappears in the post-1970 data. In this section we assess the sensitivity of the Reinhart and Rogoff results to the specific selection of post-1955 years by comparing the estimated relationship in earlier and later periods.

With the RR1955 data in the linear model, the coefficient on public debt/GDP decline by about one-third to one-half with the change from contemporaneous to future growth and cease to be significant at the 5-percent level (see Table 4). However the point estimate continues to indicate a negative relationship between public debt and future growth in the RR1955 data, and the estimates from the country fixed-effect models using RR1955 data are substantially and significantly negative. Figure 6 explores the source of the negative relationship by splitting the RR1955 data into years through 1970 and years after 1970 with a separate lowess curve for each period.

The 1955–1970 data show a much more pronounced negative relationship between growth and public debt/GDP than do the post-1970 data. Indeed, post-1970 there is essentially no relationship between public debt and future growth. Figure 6 shows that several countries went from extremely low debt to moderate debt and also experienced growth slowdowns during 1955-1970, which likely drives the negative point estimates for this period. This analysis complements Table 5, the linear regression analysis using subssamples limited to years after 1960, 1965, and 1970. For every specification, the coefficient declines steadily with the limitation to more recent years. For the post-1970 subsample, no specification yields a statistically significant result and the coefficients are generally small in magnitude.

3.6.3 Sensitivity to covariates in Cecchetti et al. (2011)

In the case of Cecchetti et al. (2011), there is a substantial divergence between the large, precise negative estimate of -0.0164 (p-val$= 0.025$) in the linear specification of the effect of Government debt on future GDP growth ("Not controlling for banking crises") reported in their Table 5, row 7, column 2 and our linear estimates in Table 4 which were ranged from zero to positive, large, and significant. Indeed in the country fixed-effect specification for forward growth in Table 4, which most closely resemble the Cecchetti et al. (2011) specification reported above, our parameter estimate is $+0.022$, i.e., positive, and significant at the 5-percent level (standard error $= 0.010$). We therefore undertook a fuller replication of Cecchetti et al. (2011)'s model. The results are presented in Table 6.

By including country fixed effects and the full set of controls from "standard growth regressions," we successfully replicated the Cecchetti et al. (2011) result of -0.016 in Table 6, column 2 of the panel titled "Controls Included, Country FE Included." But all estimates in the first three panels — every model without both controls and fixed effects — are zero or positive. That is, the Cecchetti et al. (2011) result indicating a negative effect of public debt on future GDP growth depends precisely on the inclusion of a particular set of controls and fixed effects.

Furthermore, Cecchetti et al. (2011) includes among the controls "the log of real per capita GDP at time t (to capture the 'catch-up effect' or conditional convergence of the economy to its steady state)" (p. 159). However, convergence in empirical implementations of Solow models, e.g., Mankiw et al. (1992), means convergence to the income level of the rich countries. In a model with country fixed effects, the inclusion of lagged log level of real per capita GDP effectively models convergence to the country's own average income, which is substantially different from Solow-type convergence. Thus the inclusion of once-lagged log level of real per capita GDP is not well motivated by theory. In any case, however, the model should be robust to including more than one lag of the log level of real per capita GDP, that is, robust to a generalization.

Table 6, column 1 of the panel titled "Controls Included, Country FE Included" shows the estimate of the coefficient on public debt/GDP for the CMZ model with country fixed effects and all controls except the lagged log level of real per capita GDP. The estimate of the effect of public debt on GDP growth is +0.018, i.e., positive, and significant at just above the 5-percent level (standard error = 0.009).

As we argued above, a potentially valuable control in these models is the lagged dependent variable, i.e., lagged percent change in real GDP, to capture the reverse-causal effects on public debt of persistent booms or recessions. In column 3 of the same panel we estimate the CMZ model substituting lagged percent change for the lagged log level. Estimation with a control for the lagged change in GDP, i.e., $GDP_{t-1} - GDP_{t-2}$, imposes the restriction of equal, opposite coefficients on once-lagged and twice-lagged GDP. The point estimate for the effect of public debt on GDP growth is again positive, 0.016, although significant only at the 10-percent level (standard error = 0.010). Finally in column 4, we include GDP_{t-1} and GDP_{t-2} separately, which nests both the CMZ model (with the coefficient on GDP_{t-2} restricted to zero) and the lagged dependent variable model. In column 4, the nesting model, the effect of public debt on GDP growth is estimated at zero with substantial precision.[11]

A key CMZ result thus appears to be highly fragile with respect to alternative specifications. Only the particular combination of country fixed effects, specific control variables, and once-

[11] All of these specifications, the lagged log level of real per capita GDP and the lagged percent change in real per capita GDP, potentially violate the strict exogeneity requirement in a fixed-effect model.

lagged log level of real per capita GDP yields a negative estimate for the relationship between public debt and future GDP growth. All reasonable alternative specifications, including the modest generalization of including first and second lags of log level real per capita GDP, generate substantively different results, and none of these show a negative relationship between public debt and future GDP growth.

4 Conclusions

Our re-examination of the relationship between public debt and growth in advanced economies finds little evidence to suggest a substantial, causal negative relationship. We demonstrate that there is strong indication of a reverse causal relationship from GDP growth to public debt. Indeed, contemporaneous public debt is as or more strongly correlated with GDP growth in the preceding five years than the growth in the five years in future, suggesting that weak GDP growth probably causes higher public debt. Possible mechanisms include higher deficits, i.e., reduced tax collection and increased public expenditure as well as the mechanical explanation of slow growth in the denominator of public debt/GDP.

We conclude with three main points. First, employing a range of methods, including temporal sequencing, linear models, and semi-parametric models, gives some robustness in the effort to measure the relationship between public debt and growth. The overall finding is that the effect of public debt on GDP growth is small and is zero in recent data.

Second, our findings raise serious questions about some key papers in the literature that uniformly find a negative causal relationship in recent data. If we simply consider a bivariate relationship between forward growth and public debt, there is essentially no relationship between these two since 1970, the period examined in Cecchetti et al. (2011), Woo and Kumar (2015), and Checherita-Westphal and Rother (2012). The conclusions in those papers of a negative relationship result from sensitive sets of controls or tests for trend breaks that are sensitive to small samples, outliers and parametric choices. Moreover, despite the consistent finding of an important growth threshold around 90 percent with alternative methods and samples employed by Reinhart and Rogoff (2010), Cecchetti et al. (2011), Checherita-Westphal and Rother (2012), and Woo and Kumar (2015), our non- and semi-parametric plots strongly suggest that there is no threshold at elevated levels of public debt. Our findings underscore the importance of looking at the data themselves rather than relying on opaque parametric tests to determine how public debt affects growth.

Finally, many policy decisions to confront public debt via austerity have hinged on the presumption of a threshold. There is no evidence of a public debt threshold above which growth is substantially reduced in any of the data, using any method.

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Table 1: *Countries included, by data source*

	Full Sample (Start year–2011)	RR1955 (1955–2003)	CMZ (1985–2003)	CWR (1975–2003)	WK (1975–2003)
Australia	1880	✓	✓		✓
Austria	1880	✓	✓	✓	✓
Belgium	1880	✓	✓	✓	✓
Canada	1880	✓	✓		✓
Denmark	1880	✓	✓		✓
Finland	1914	✓	✓	✓	
France	1880	✓	✓	✓	✓
Germany	1880	✓	✓	✓	✓
Greece	1884	✓	✓	✓	✓
Iceland	1950				✓
Ireland	1929	✓		✓	
Italy	1880	✓	✓	✓	✓
Japan	1880	✓	✓		✓
Luxembourg				✓	
Netherlands	1880	✓	✓	✓	✓
New Zealand	1880	✓			
Norway	1880	✓	✓		
Portugal	1880	✓	✓	✓	✓
Spain	1880	✓	✓	✓	✓
Sweden	1880	✓	✓		✓
Switzerland	1899				✓
UK	1880	✓	✓		✓
USA	1880	✓	✓		✓

Notes. The table reports the countries and years used in our analysis. ✓ indicates availability of data for the set of years listed at the top of the column; otherwise a list of years reports missing data for the country.

Table 2: *Public Debt and Growth: Regression Results for Full Sample*

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Dep. Var.: Current growth in real GDP; No additional controls							
Public Debt/GDP	-0.015***	-0.010***	-0.012***	-0.004	-0.004	-0.014***	-0.000
	(0.004)	(0.003)	(0.003)	(0.004)	(0.003)	(0.004)	(0.005)
Observations	2,558	2,558	2,500	2,362	2,344	2,558	2,362
Dep. Var.: Future 5-year growth in real GDP; No additional controls							
Public Debt/GDP	-0.011***	-0.008***	-0.008***	-0.007**	-0.007*	-0.007**	-0.004
	(0.004)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.005)
Observations	2,419	2,419	2,361	2,226	2,208	2,419	2,226
Dep. Var.: Future 5-year growth in real GDP; No additional controls; Restricted Sample							
Public Debt/GDP	-0.010***	-0.007**	-0.007**	-0.008**	-0.008**	0.003	0.000
	(0.003)	(0.003)	(0.003)	(0.004)	(0.004)	(0.007)	(0.007)
Observations	500	500	500	500	500	500	500
Dep. Var.: Future 5-year growth in real GDP; with added controls							
Public Debt/GDP	-0.007*	-0.006***	-0.006***	-0.008***	-0.009***	-0.001	-0.007
	(0.004)	(0.002)	(0.002)	(0.003)	(0.003)	(0.004)	(0.005)
Observations	500	500	500	500	500	500	500
Year FE		Y	Y	Y	Y	Y	Y
Control: 1yr Lag Gwth			Y		Y		
IV: 5yr Lag Debt				Y	Y		Y
Country FE						Y	Y

Notes. Current growth and future 5-year average growth in real per capita GDP explained by the public debt-to-GDP ratio. Robust standard errors, clustered by country, in parentheses below parameter estimates. Significance levels are as follows: * p<0.10, ** p<0.05, *** p<0.01. FS1880 refers to the Full Sample of countries, with some starting in 1880 and all extending through 2011.

Table 3: *Public Debt and Future Growth in real GDP and in real TFP*

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Dep. Var. = Future 5-year growth in real GDP							
Public Debt/GDP	-0.019***	-0.012***	-0.010**	-0.011	-0.011	-0.016***	-0.013
	(0.006)	(0.004)	(0.004)	(0.007)	(0.007)	(0.005)	(0.010)
Observations	1,182	1,182	1,142	1,154	1,128	1,182	1,154
Dep. Var. = Future 5-year growth in real TFP							
Public Debt/GDP	-0.012***	-0.008***	-0.007**	-0.009**	-0.008**	-0.009***	-0.011**
	(0.004)	(0.003)	(0.003)	(0.004)	(0.004)	(0.003)	(0.005)
Observations	1,182	1,182	1,142	1,154	1,128	1,182	1,154
Year FE		Y	Y	Y	Y	Y	Y
Control: 1yr Lag Gwth			Y		Y		
IV: 5yr Lag Debt				Y	Y		Y
Country FE						Y	Y

Notes. Row 1: Dep Var = Future 5-year growth in real GDP; no additional controls. Row 2: Dep Var = Future 5-year growth in real TFP; no additional controls. Estimation samples are restricted to the observations that have non-missing values for both of the dependent variables. Robust standard errors,clustered by country, in parentheses below parameter estimates. Significance levels are as follows: * p<0.1, ** p<0.05, *** p<0.01.

Table 4: *Public Debt and Future Growth: Regression Results by Dataset*

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Full Sample: 22 countries (variable–2011)							
Public Debt/GDP	-0.011***	-0.008***	-0.008***	-0.007**	-0.007*	-0.007**	-0.004
	(0.004)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.005)
Observations	2419	2419	2361	2226	2208	2419	2226
RR1955: 20 countries (1955–2003)							
Public Debt/GDP	-0.013*	-0.011*	-0.007	-0.004	-0.003	-0.019***	-0.012
	(0.007)	(0.006)	(0.006)	(0.010)	(0.009)	(0.007)	(0.009)
Observations	929	929	905	825	822	929	825
CWR: 12 countries (1975–2003)							
Public Debt/GDP	0.004	-0.002	0.001	0.004	0.004	0.018	0.056***
	(0.008)	(0.007)	(0.007)	(0.012)	(0.009)	(0.015)	(0.015)
Observations	335	335	323	275	275	335	275
CMZ: 18 countries (1985–2003)							
Public Debt/GDP	0.003	-0.000	-0.000	-0.002	-0.002	0.019*	0.074*
	(0.005)	(0.004)	(0.004)	(0.005)	(0.005)	(0.010)	(0.040)
Observations	342	342	324	252	252	342	252
WK: 18 countries (1975–2003)							
Public Debt/GDP	0.002	-0.001	-0.001	-0.001	-0.001	0.006	0.016
	(0.004)	(0.003)	(0.003)	(0.004)	(0.004)	(0.011)	(0.021)
Observations	522	522	504	432	432	522	432
Year FE		Y	Y	Y	Y	Y	Y
Control: 1yr Lag Gwth			Y		Y		
IV: 5yr Lag Debt				Y	Y		Y
Country FE						Y	Y

Notes. Future 5-year average growth in real GDP explained by the public debt-to-GDP ratio. Robust standard errors,clustered by country, in parentheses below parameter estimates. Significance levels are as follows: * p<0.1, ** p<0.05, *** p<0.01. For references to the datasets see Section 2.3.

Table 5: *Public Debt and Future Growth: Robustness to Using Alternative Start Dates*

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Full Sample							
Public Debt/GDP	-0.011***	-0.008***	-0.008***	-0.007*	-0.007**	-0.007**	-0.004
	(0.004)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.005)
Observations	2419	2419	2361	2208	2226	2419	2226
Year>=1960							
Public Debt/GDP	-0.017***	-0.008*	-0.006	-0.006	-0.007	-0.014**	-0.012
	(0.005)	(0.004)	(0.005)	(0.008)	(0.008)	(0.006)	(0.009)
Observations	1034	1034	1011	924	924	1034	924
Year>=1965							
Public Debt/GDP	-0.012***	-0.006	-0.005	-0.002	-0.002	-0.010**	-0.004
	(0.005)	(0.004)	(0.004)	(0.008)	(0.008)	(0.005)	(0.011)
Observations	924	924	902	814	814	924	814
Year>=1970							
Public Debt/GDP	-0.004	-0.004	-0.002	-0.001	-0.001	-0.004	0.004
	(0.005)	(0.004)	(0.005)	(0.008)	(0.008)	(0.006)	(0.013)
Observations	814	814	792	704	704	814	704
Year Fixed Effects		Y	Y	Y	Y	Y	Y
Control: 1yr Lag Gwth			Y	Y			
IV: 5yr Lag Debt				Y	Y		Y
Country Fixed Effects						Y	Y

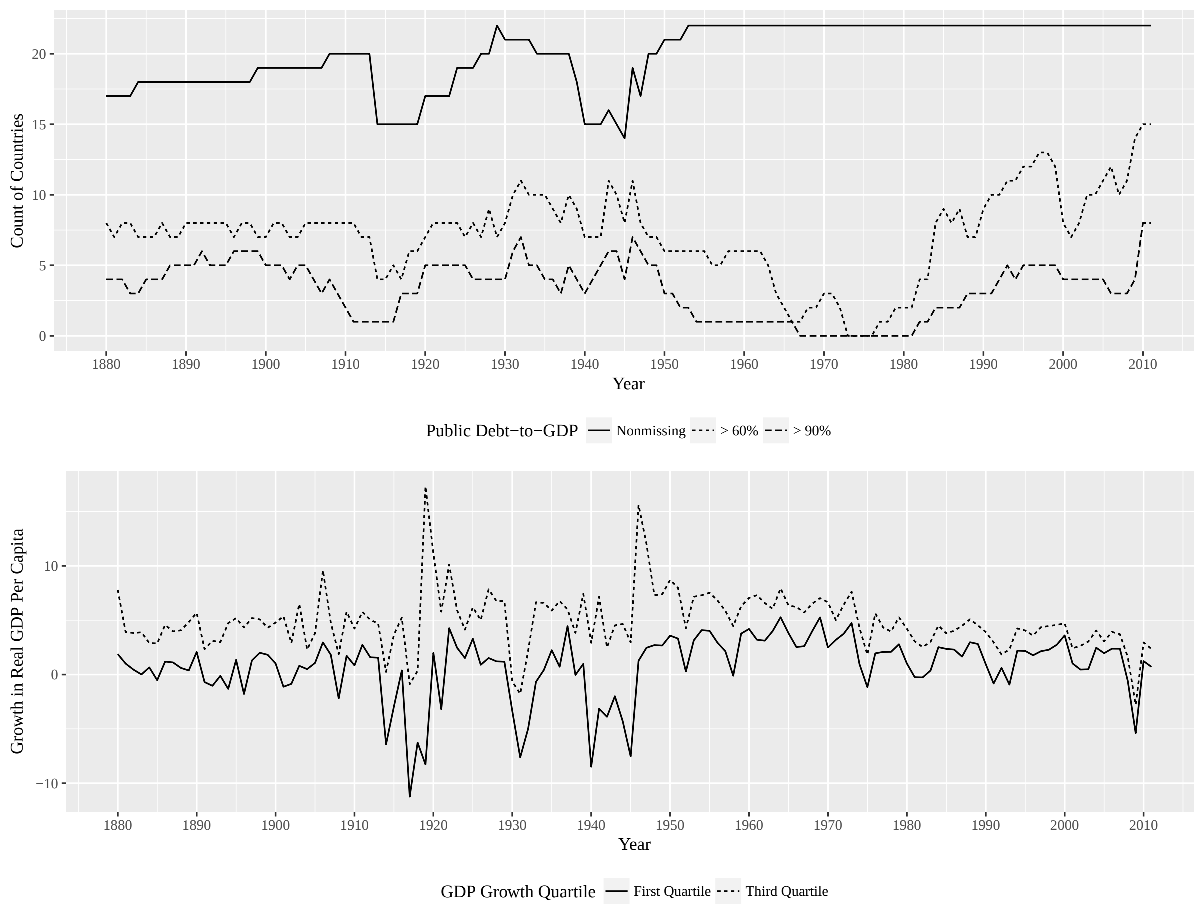
Notes. Future 5-year average growth in real GDP explained by the public debt-to-GDP ratio. Robust standard errors, clustered by country, in parentheses below parameter estimates. Significance levels are as follows: * $p<0.1$, ** $p<0.05$, *** $p<0.01$.

Table 6: *Replication of Regression Results in Cecchetti et al. (2011)*

	(1)	(2)	(3)	(4)
Controls Excluded, Country FE Excluded				
Public Debt/GDP	-0.000	-0.001	-0.000	-0.000
	(0.004)	(0.004)	(0.004)	(0.004)
Observations	342	342	324	306
Controls Excluded, Country FE Included				
Public Debt/GDP	0.019*	-0.007	0.020*	-0.000
	(0.010)	(0.012)	(0.010)	(0.004)
Observations	342	342	324	306
Controls Included, Country FE Excluded				
Public Debt/GDP	0.000	-0.002	0.001	-0.000
	(0.006)	(0.005)	(0.006)	(0.004)
Observations	342	342	324	306
Controls Included, Country FE Included				
Public Debt/GDP	0.009	-0.016***	0.008	-0.000
	(0.008)	(0.006)	(0.007)	(0.004)
Observations	342	342	324	306
Year FE	Y	Y	Y	Y
1-Lag GDP		Y		Y
2 Lags of GDP				Y
1-Lag Growth of GDP			Y	

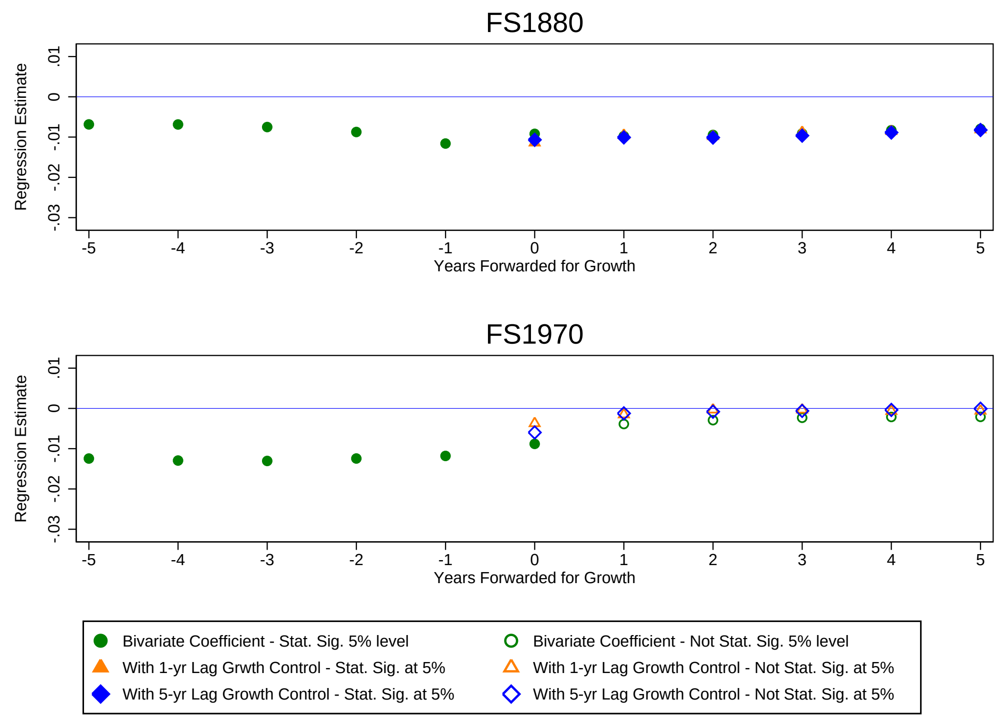
Notes. Future 5-year average growth in real per capita GDP explained by the public debt-to-GDP ratio. Additional controls include the variables used by CMZ: savings, growth of population, years of schooling, trade openness, the rate of inflation, dependency ratio, and liquid liabilities. Robust standard errors, clustered by country, appear in parentheses below parameter estimates. Significance levels are as follows: * $p<0.1$, ** $p<0.05$, *** $p<0.01$. Standard errors in parentheses.

Figure 1: *Prevalence of high public debt and range of GDP growth, 1880-2011*



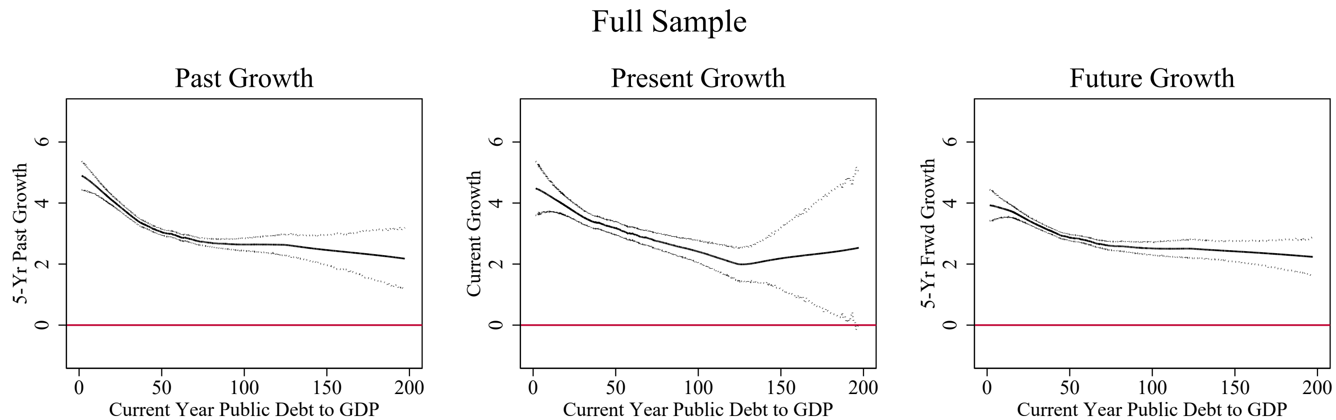
Notes. The top panel shows the count of countries out of the 22 countries in the full sample with public debt-to-GDP ratio above 60 percent or above 90 percent between 1880 and 2011. The bottom panel the interquartile range of real per capita GDP growth among the 22 countries in the Full Sample between 1880 and 2011.

Figure 2: *Coefficient on Public Debt-to-GDP Ratio with Alternative Windows*



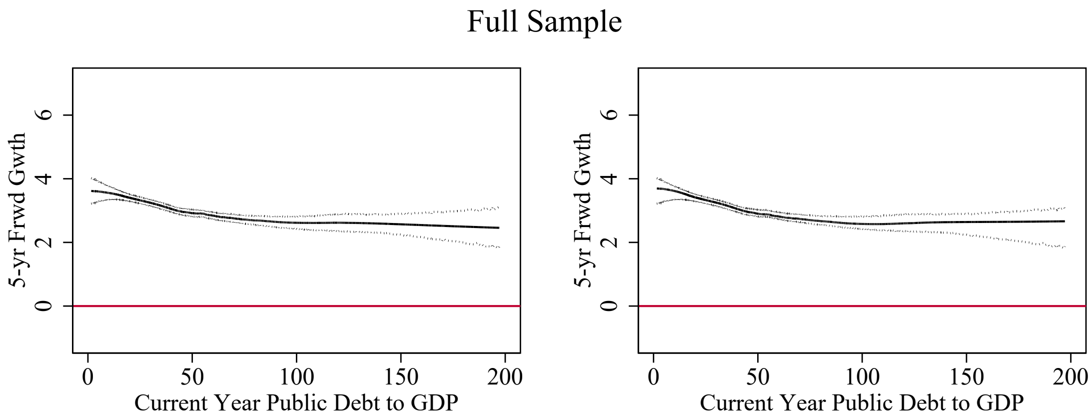
Notes. The plot shows coefficients from regressions of real GDP growth on current public debt when the time window for averaging GDP growth varies from -5 to $+5$ years. When the dependent variable is contemporaneous or forwarded growth, alternative controls for lagged growth include no control and controls for 1- and 5-year lagged growth. All specifications include year dummies. Filled markers indicate statistical significance at the 5 percent level with country-clustered standard errors. FS1880 refers to the full sample for all available years, FS1970 to the full sample limited to 1970-2011. For references to the datasets see Section 2.3.

Figure 3: *Public Debt-Growth Lowess Plots*



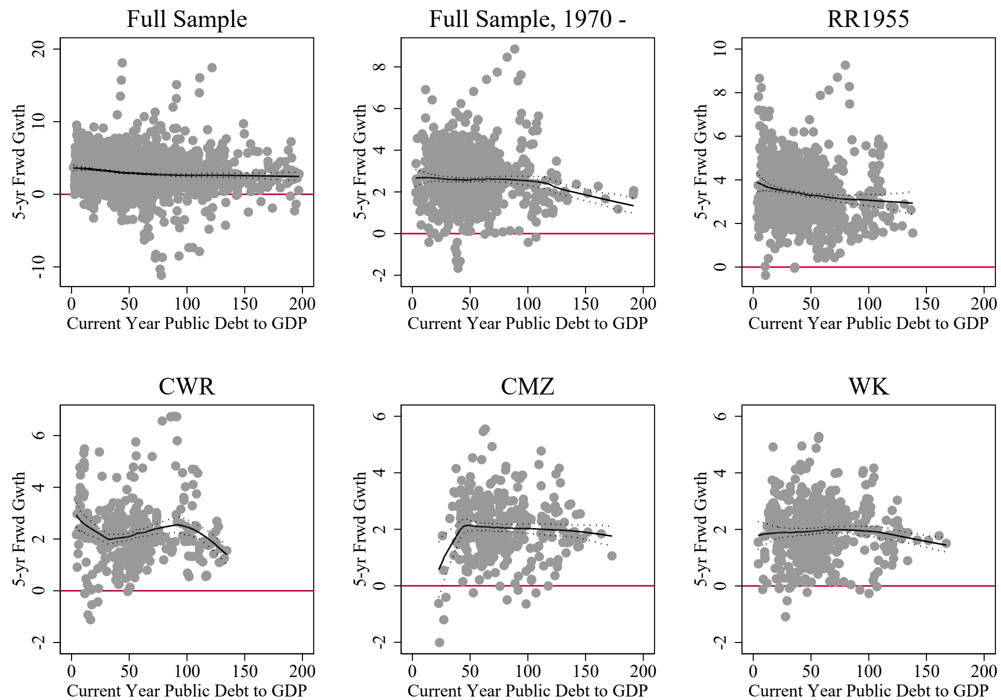
Notes. Lowess plots for the bivariate relationship between public debt and growth. For references to the datasets see Section 2.3. In each row, the relationship between contemporary public debt and average real GDP growth for the preceding five years is show in the left panel, between contemporary public debt and contemporary real GDP growth in the center panel, between average real GDP growth for the following five years in the right panel. Bootstrapped confidence intervals based on 250 repetitions.

Figure 4: *Public Debt-Future Growth Partial Linear Regression*



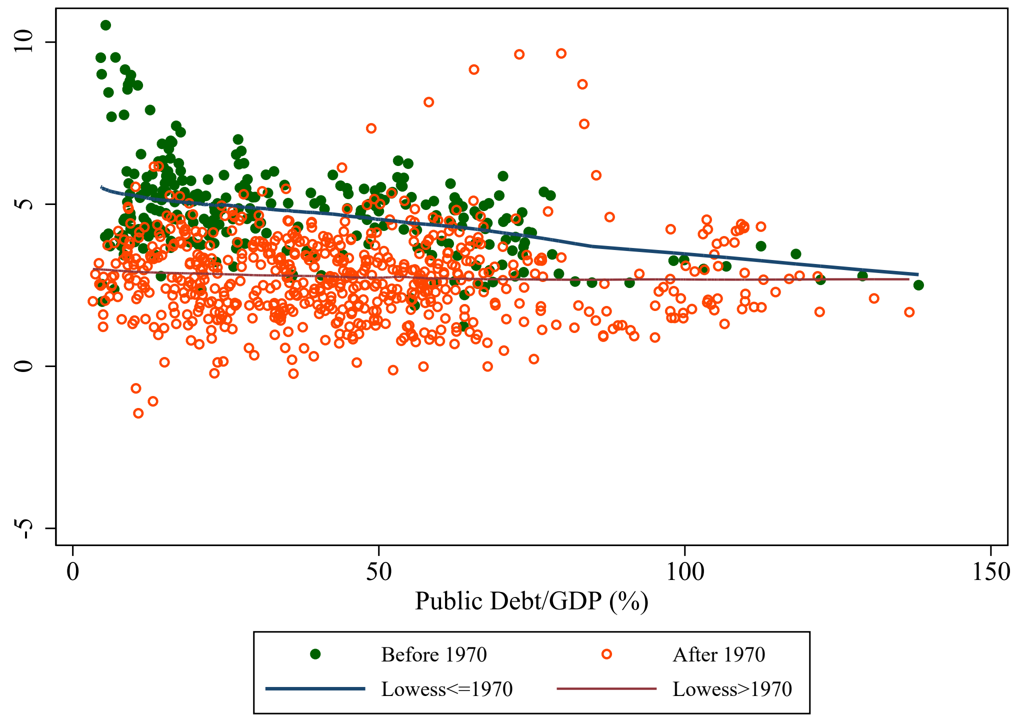
Notes. Partial Linear Regression Residual Lowess plots for the relationship between public debt and five-year future average real GDP growth. Left panel shows the model with 1-lagged growth and year fixed effects; right panel shows the model with country and year fixed effects. FS1880 refers to the Full Sample of countries with some starting in 1880 and all ending in 2011. Bootstrapped confidence intervals based on 250 repetitions.

Figure 5: *Public Debt-Future Growth Partial Linear Regression Controlling for Lagged Growth with Scatterplot*



Notes. Partial Linear Regression Residual Lowess plots for the relationship between public debt and five-year future average real GDP growth controlling for one-year lagged real GDP growth and including year dummies. For references to the datasets see Section 2.3. Bootstrapped confidence intervals based on 250 repetitions.

Figure 6: *Public Debt-Future Growth Partial Linear Regression for RR1955, by pre/post-1970*



Notes. Partial Linear Regression Residual Lowess plots for the relationship between public debt and five-year future average real GDP growth controlling for one-year lagged real GDP growth and including year dummies. Future For references to the datasets see Section 2.3.

Online Appendix (Not for Publication)

This Online Appendix presents:

A. A formal derivation of conditions under which the bias in the estimate of the impact of debt on growth is reduced by (1) using forwarded growth, (2) controlling for lagged dependent variable, and (3) instrumenting public debt with lagged public debt.

B. Description of our dataset

C. A schematic of the spline specification used in Cecchetti et al. (2011) and Woo and Kumar (2015) in Figure D.1

A Bias in Estimation of the Debt-Growth Relationship

A.1 Basic Setup

Appendix A establishes specific conditions for the time-series properties of the endogenous growth and debt process that illustrate when each of the approaches works best and how to interpret concurrent or divergent results with the three approaches. The Lagged Growth Control and Average Forward Growth approaches focus identification on different parts of the data-generating process. Controlling for lagged growth focuses identification of the effect of debt on innovations in growth, i.e., persistent growth is controlled for via the inclusion of the lag. Average Forward Growth focuses identification on the persistent portion of growth, i.e., the accumulation of growth over a five-year period. These alternative foci imply different responses of the estimator to alternative values of $(\rho_u(k), \rho_v(k))$, where $\rho_u(k)$ and $\rho_v(k)$ denote the k-th order autocorrelation coefficient for u and v, respectively. In Appendix A, we illustrate these arguments for the case when the error terms in the growth and debt equations follow stable AR(1) processes, with AR coefficients ϕ_u and $\phi_v$respectively.

The Lagged Growth Control and Average Forward Growth estimators have the convenient property that they reduce bias (or are biased towards zero) under opposite assumptions about ϕ_u. Average Forward Growth, focused on the persistent component, performs better whenϕ_u is relatively low and will be biased toward zero only if ϕ_u is relatively high. The Lagged Growth Control, focused on innovations, performs better when ϕ_u is relatively high and will be biased toward zero only if ϕ_u is relatively low.

This combination means that it cannot be that both of them are biased towards zero (under the data-generating process). If, for example, both reduce the magnitude of the estimate vis-à-vis the baseline OLS estimate, then the reduced magnitude constitutes an improvement on the baseline estimate, and is not a result of specific conditions on (ϕ_u, ϕ_v) that bias the estimate toward zero. The similarity of the estimates for the Lagged Growth Control and the Average Forward Growth specification in the empirical analysis imply that the likely values of (ϕ_u, ϕ_v) are in the intersection of the zones where each strategy reduces bias.

Let g_{it} and D_{it} denote de-meaned growth (annual growth rate of real gross domestic product) and public debt (ratio of public debt to nominal gross domestic product) respectively. The structural model of bi-directional causal relationships between growth and debt, i.e., the "true" data generating process (DGP), is given by the following simultaneous equation model

$$D_{it} = a_1 g_{it} + v_{it} \tag{5}$$

and

$$g_{it} = b_1 D_{it} + u_{it} \tag{6}$$

where the structural errors are distributed as $v_{it} \sim (0, \sigma_v^2)$ with k^{th}-order autocorrelation coefficient $\rho_v(k)$, $u_{it} \sim (0, \sigma_u^2)$ with k^{th}-order autocorrelation coefficient $\rho_u(k)$, and for all integers $k = 0, 1, 2, \ldots$, $\mathbb{E}(v_{i,t+k} u_{i,t+k}) = 0$, where $\mathbb{E}(y)$ refers to the expected value of the random variable, y. We formalize the reverse causality problem with the following two equations. The first is growth, which depends on public debt,

$$g_{it} = D_{it} b_1 + u_{it} \tag{7}$$

in which g_{it} represents the growth rate of GDP for country i in year t, D_{it} represents the stock of public debt, b_1 is a parameter, and u_{it} represents the error term and includes all other controls or contributors to GDP growth. The second equation describes the evolution of public debt,

$$D_{it} = g_{it} a_1 + v_{it} \tag{8}$$

which introduces the additional parameter a_1 and error term v_{it}.

Reverse causality biases the OLS estimate of b_1 with the asymptotic bias given by

$$\text{plim } \hat{b}_1 - b_1 = \frac{a_1(1 - a_1 b_1)}{a_1^2 + \lambda} \tag{9}$$

where $\lambda = \frac{\sigma_v^2}{\sigma_u^2}$ is the ratio of the variance of the error terms. If b_1 is reasonably small and $a_1 < 0$, then the bias is negative, i.e., public debt is estimated to be worse for GDP growth than it actually is.

Let us suppose reverse causality, i.e., $a_1 < 0$, and no forward causality, i.e., $b_1 = 0$. Autocorrelation in the error term for the growth equation means that a negative shock to growth will both be persistent in itself, i.e., recessions linger, and the shock is passed into the public debt process for a long time, which raises public debt. In a contemporaneous regression, autocorrelation in the growth equation will erroneously lead to the conclusion that public debt is bad for growth, i.e., $\widehat{b_1} < 0$. We show below that averaging growth over several future periods reduces the bias. Autocorrelation in the error term for public debt means that a positive shock to public debt persist for a long time. Under the same supposition about the true value of the parameters, b_1 will be properly estimated.

The reduced form model corresponding to the structural model in (5) and (6) is given by

$$D_{it} = \left\{ \frac{a_1}{1 - a_1 b_1} \right\} u_{it} + \left\{ \frac{1}{1 - a_1 b_1} \right\} v_{it} \tag{10}$$

and

$$g_{it} = \left\{ \frac{1}{1 - a_1 b_1} \right\} u_{it} + \left\{ \frac{b_1}{1 - a_1 b_1} \right\} v_{it}. \tag{11}$$

We use the following notation: $\operatorname{plim} z_n$ refers to the probability limit of the random variable z_n when the appropriate index, n, goes to infinity; $|x|$ refers to the absolute value of a real number x. Our maintained assumption throughout this paper is stated as

Assumption 1. *For the model in (5) and (6), $a_1 < 0$, $b_1 < 0$, and $a_1 b_1 < 1$.*

Let $\hat{b}_1$ denote the OLS estimator of b_1 in (6) and let $\lambda = (\sigma_v^2/\sigma_u^2)$; then, assuming that the relevant exogeneity conditions of the structural errors in (5) and (6) hold, we have

$$\operatorname{plim} \hat{b}_1 = \frac{\mathbb{E}(D_{it} g_{it})}{\mathbb{E}(D_{it}^2)} = \frac{a_1 + b_1 \lambda}{a_1^2 + \lambda}. \tag{12}$$

The estimator $\hat{b}_1$ in the above equation is indexed by the sample size and the probability limit is computed with the sample size approaching infinity. But here, and below, we suppress explicit indexing with the sample size for notational simplicity.

Using the above expression for the OLS estimator of b_1, we can see that the bias of the OLS estimator (due to reverse causality) is given by

$$\operatorname{plim} \hat{b}_1 - b_1 = \frac{a_1(1 - a_1 b_1)}{a_1^2 + \lambda} < 0, \tag{13}$$

where the inequality holds because of assumption 1. Hence, bias in the OLS estimator is negative, i.e., the “true” impact of debt on growth is smaller in magnitude than what the

OLS estimator shows.

A.2 Conditions for Bias Reduction

Let $\widetilde{b}_1$ be an estimator of b_1 arrived at using some specification other than (6) or some method other than OLS. We are interested in understanding the conditions under which this alternative specification (or method) takes us closer to the "true" parameter b_1 *from below*, i.e., while the direction of asymptotic bias in $\widetilde{b}_1$ remains the same as for the OLS estimator $\hat{b}_1$, i.e., negative, its magnitude is reduced. This would be ensured when $\operatorname{plim} \widetilde{b}_1$ is bounded from below by the probability limit of the OLS estimator and from above by the "true" value of the parameter, i.e.,

$$\operatorname{plim} \hat{b}_1 < \operatorname{plim} \widetilde{b}_1 < b_1. \tag{14}$$

We limit ourselves to downward biased estimators because our analysis suggests that the effect of debt on growth is less negative than claimed in the extant literature. The opposite case, where estimators could be upward biased, is of limited interest to us.

While we derive conditions for bias reduction in terms of the correlation coefficients of the two error terms - the error term in the growth and in the debt equation, we illustrate these conditions for a specific case: when the error terms for the growth and debt equations follow stable AR(1) processes with AR coefficients, ϕ_u and ϕ_v respectively. For each method, we illustrate the condition for bias reduction by depicting the locus of (ϕ_u, ϕ_v) on the unit square for which bias is reduced, i.e., the approach improves on OLS if (ϕ_u, ϕ_v) falls in the shaded range. We generate Figure C.1 with plausible values for a, b, and λ: $a = -20$, which implies that a severe recession with growth of -4 percent per year generates debt of 80 percent; $b = -0.012$, which implies that a public debt-to-GDP ratio of 100 percent generates a severe recession with growth of -1.2 percent per year; and $\lambda = 16$, under the supposition that a standard deviation of public debt is 8 percentage points of GDP and the standard deviation of GDP growth is 2 percentage points.

B Bias Reduction in Alternative Strategies

We will use the general conditions listed above in (14) to derive *sufficient* conditions for reduction in the magnitude of the negative bias in alternative estimation strategies.

B.1 k-period Forwarding of Growth

Let k be any positive integer. Many papers in the emerging literature on the growth-debt relationship use forwarding of the dependent variable (growth) to reduce the bias due to reverse causality. While it might be intuitively clear that forwarding reduces the bias, we would like to investigate the question more rigorously here and ask: under what conditions does k-period forwarding of the dependent variable in (6) reduce the bias in the OLS estimator?

To proceed, note that with k-period forwarding of the dependent variable, the following equation is estimated

$$g_{i,t+k} = b_{11} D_{it} + \varepsilon_{it}. \tag{15}$$

instead of (6). Let $\hat{b}_{11}$ be the OLS estimator of b_{11} in (15); then we have

$$\operatorname{plim} \hat{b}_{11} = \frac{\mathbb{E}(D_{it} g_{t+k})}{\mathbb{E}(D_{it}^2)} = \frac{a_1 \rho_u(k) + b_1 \lambda \rho_v(k)}{a_1^2 + \lambda} \tag{16}$$

where the second equality comes from using the expression for D_{it} and $g_{i,t+k}$ corresponding to the "true" DGP in (5) and (6), and

$$\rho_v(k) = \frac{\mathbb{E}(v_{it} v_{i,t+k})}{\sigma_v^2}$$

is the k-th order autocorrelation coefficient in v_{it}, and

$$\rho_u(k) = \frac{\mathbb{E}(u_{it}u_{i,t+k})}{\sigma_u^2}$$

is the k-th order autocorrelation coefficient in u_{it}. We assume that the structural errors, u_{it} and v_{it}, have non-negative autocorrelation coefficients and state this as

Assumption 2. *For all* $j = 0, 1, 2, \ldots$, $\rho_v(j) \geq 0$ *and* $\rho_u(j) \geq 0$.

Proposition 1. *If assumption 1 and 2 holds, and*

$$a_1 \left\{\rho_u(k) - a_1 b_1\right\} < b_1 \lambda \left\{1 - \rho_v(k)\right\} \tag{17}$$

then k-period forwarding reduces bias from below, i.e., $\operatorname{plim} \hat{b}_1 < \operatorname{plim} \hat{b}_{11} < b_1$.

Proof. Since $\lambda > 0$, $0 \geq \rho_u(k) \geq 1$, $0 \geq \rho_v(k) \geq 1$, $a_1 < 0$ and $b_1 < 0$, we have $a_1\rho_u(k) > a_1$ and $b_1\lambda\rho_v(k) > b_1\lambda$, so that

$$a_1 + b_1\lambda < a_1\rho_u(k) + b_1\lambda\rho_v(k)$$

and hence that

$$\operatorname{plim} \hat{b}_1 = \frac{a_1 + b_1\lambda}{a_1^2 + \lambda} < \frac{a_1\rho_u(k) + b_1\lambda\rho_v(k)}{a_1^2 + \lambda} = \operatorname{plim} \hat{b}_{11}$$

using the expressions in (12) and (16). Thus, the probability limit of the alternative estimator is bounded from below by the OLS estimator. To see the upper bound, note that if

$$a_1 \left\{\rho_u(k) - a_1 b_1\right\} < b_1 \lambda \left\{1 - \rho_v(k)\right\}$$

then

$$a_1\rho_u(k) + b_1\lambda\rho_v(k) < b_1 a_1^2 + b_1\lambda$$

so that

$$\text{plim}\,\hat{b}_{11} = \frac{a_1\rho_u(k) + b_1\lambda\rho_v(k)}{a_1^2 + \lambda} < b_1.$$

This completes the proof. □

To get some intuitive understanding of the result in proposition 1, let us compare the expressions for the OLS estimator in (12) and the k-period forwarded estimator in (16). Note two extreme cases regarding the magnitudes of the autocorrelations in the structural errors: (a) when the autocorrelations tend towards unity from below, i.e., $(\rho_u(k), \rho_v(k)) \uparrow (1, 1)$, the k-period forwarded estimator collapses to the OLS estimator; (b) when the autocorrelations tend towards zero from above, i.e., $(\rho_u(k), \rho_v(k)) \downarrow (0, 0)$, the k-period forwarded estimator tends towards zero (which leads to positive bias, because the true parameter is negative, i.e., $b_1 < 0$). Hence the k-period forwarded estimator reduces the magnitude of the negative bias when the autocorrelations in the two structural errors are bounded away sufficiently strongly from the extreme values of 0 and 1, i.e., the errors need to be correlated with its past values but not too strongly. The condition in (17) gives the precise way in which this bounding away is needed to achieve reduction in bias from below. Since, $0 < \rho_u(k), \rho_v(k) < 1$, combinations of these two autocorrelations, i.e., $(\rho_u(k), \rho_v(k))$, fall in the unit square on the positive part of the 2D plane. The condition in (17) defines the subset of this unit square which would be consistent with reduction in the magnitude of the negative bias.

B.2 k-period Average Forwarding

For any variable x_{it}, let k-period average forwarding be denoted by $\widetilde{x}_{it}$, i.e.,

$$\widetilde{x}_{it} = \frac{1}{k}\sum_{j=0}^{k-1} x_{i,t+j}.$$

With k-period average forwarding of the dependent variable, instead of (6), the following equation is estimated,

$$\widetilde{g}_{i,t+k} = b_{12} D_{it} + \varepsilon_{it}. \tag{18}$$

Let $\hat{b}_{12}$ be the OLS estimator of b_{12} in (18), and let

$$\widetilde{\rho}_v(k) = (1/k)\left\{\sum_{j=0}^{k-1} \rho_v(j)\right\} = (1/k)\left\{1 + \sum_{j=1}^{k-1} \rho_v(j)\right\},$$

and

$$\widetilde{\rho}_u(k) = (1/k)\left\{\sum_{j=0}^{k-1} \rho_u(j)\right\} = (1/k)\left\{1 + \sum_{j=1}^{k-1} \rho_u(j)\right\};$$

then we have

$$\operatorname{plim} \hat{b}_{12} = \frac{\mathbb{E}(D_{it}\widetilde{g}_{i,t+k})}{\mathbb{E}(D_{it}^2)} = \frac{a_1 \widetilde{\rho}_v(k) + b_1 \lambda \widetilde{\rho}_u(k)}{a_1^2 + \lambda} \tag{19}$$

Proposition 2. *If assumption 1 holds, and*

$$a_1 \left\{\widetilde{\rho}_u(k) - a_1 b_1\right\} < b_1 \lambda \left\{1 - \widetilde{\rho}_v(k)\right\} \tag{20}$$

then k-period average forwarding reduces bias from below, i.e., $\operatorname{plim} \hat{b}_1 < \operatorname{plim} \hat{b}_{12} < b_1$.

Proof. The proof follows the same logic as the proof of the previous proposition when $\rho_v(k)$ and $\rho_u(k)$ are replaced with $\widetilde{\rho}_v(k)$ and $\widetilde{\rho}_u(k)$, respectively. □

Equation 19 gives insight into the effectiveness of the approach of average forwarding for recovering the true value of b_1. If $\rho_u(k)$ and $\rho_v(k)$ are both close to one, the estimate of b_1 based on k-period average forwarding approaches the OLS estimate, $\operatorname{plim} \hat{b}_{12} = plim\hat{b}_1$, and average forwarding is, like OLS, negatively biased. If $\rho_u(k)$ and $\rho_v(k)$ are both close to zero, the estimate of b_1 based on k-period average forwarding approaches zero, i.e., $\operatorname{plim} \hat{b}_{12} = 0$, and assuming $b1 < 0$, average forwarding is positively biased. This result is not surprising because the average forwarding estimator focuses on the persistent component of growth in

GDP. If there is little persistent component, i.e., $\rho_u(k) = 0$, then the approach does not yield useful results. For intermediate values of $\rho_u(k)$ and $\rho_v(k)$, average forwarding will recover the true value of b_1.

We can illustrate these conditions for bias reduction with a specific example. Suppose u and v both follow AR(1) processes so that

$u_{it} = \phi_u u_{i,t-1} + \epsilon^1_{it}$ and $v_{it} = \phi_v v_{i,t-1} + \epsilon^2_{it}$, where $|\phi_u| < 1$, and $|\phi_v| < 1$ so that the AR(1) processes are stable. Then, if we use 5-year average forwarding, the condition for bias reduction, 20, becomes $a_1 \left\{ \frac{1-\phi_u^5}{5(1-\phi_u)} - a_1 b_1 \right\} - b_1 \lambda \left\{ 1 - \frac{1-\phi_v^5}{5(1-\phi_v)} \right\} < 0$.

In the upper left-hand panel of Figure C.1, we plot the function $F(\phi_u, \phi_v) = a_1 \left\{ \frac{1-\phi_u^5}{5(1-\phi_u)} - a_1 b_1 \right\} - b_1 \lambda \left\{ 1 - \frac{1-\phi_v^5}{5(1-\phi_v)} \right\}$ on the unit square, using the following parameter values $a_1 = -20, b_1 = -0.012, \lambda = 16$, and indicate the region on the unit square where the value of the function is negative. This area represents the combination of (ϕ_u, ϕ_v) for which the estimator based on the use of Average Forward Growth as the dependent variable improves on the OLS estimate (with contemporaneous growth as the dependent variable).

B.3 Lagged Debt as an IV for Current Debt

In this case, we estimate (6) using $D_{i,t-5}$ as an instrument for D_{it}.[12] Let $\hat{b}_{13}$ be the IV estimator for b_1 in (6). We have,

$$\operatorname{plim} \hat{b}_{13} = \frac{\mathbb{E}(g_{it} D_{i,t-5})}{\mathbb{E}(D_{it} D_{i,t-5})} = \frac{a_1 \rho_u(5) + b_1 \lambda \rho_v(5)}{a_1^2 \rho_u(5) + \lambda \rho_v(5)} = \frac{a_1 + b_1 \lambda \omega(5)}{a_1^2 + \lambda \omega(5)} \tag{21}$$

where $\omega(5) = (\rho_v(5)/\rho_u(5))$ is the ratio of the autocorrelation coefficients of order 5.

Proposition 3. *If assumption 1 and 2 holds, and* $\omega(5) > 1$, *then using* $D_{i,t-5}$ *as an instrument for* D_{it} *in (6) reduces the bias from below, i.e.,* $\operatorname{plim} \hat{b}_1 < \operatorname{plim} \hat{b}_{13} < b_1$.

[12] We have used the 5-th lag because that has been used in the literature. There is no reason why one could not use any other lag. The results derived here are valid for any lag, $k = 1, 2, 3, \ldots$.

Proof. Note that the upper bound is always satisfied, because $a_1 < 0$ and $a_1 b_1 < 1$ implies that

$$\text{plim}\,\hat{b}_{13} = \frac{a_1 + b_1\lambda\omega(5)}{a_1^2 + \lambda\omega(5)} < b_1.$$

To see that the lower bound is satisfied note that $\omega(5) > 1$ implies

$$a_1\lambda\left\{1 - \omega(5)\right\}(1 - a_1 b_1) > 0$$

so that

$$a_1\lambda\left\{1 - \omega(5)\right\} > \lambda a_1^2 b_1\left\{1 - \omega(5)\right\}.$$

Adding $a_1^3 + b_1\lambda^2\omega(5)$ to both sides of the above we get

$$\left\{a_1 + b_1\lambda\omega(5)\right\}\left\{a_1^2 + \lambda\right\} > \left\{a_1 + b_1\lambda\right\}\left\{a_1^2 + \lambda\omega(5)\right\}$$

which shows that

$$\text{plim}\,\hat{b}_1 = \frac{a_1 + b_1\lambda}{a_1^2 + \lambda} < \frac{a_1 + b_1\lambda\omega(5)}{a_1^2 + \lambda\omega(5)} = \text{plim}\,\hat{b}_{13} \tag{22}$$

This completes the proof. □

An intuitive understanding of the result in proposition 3 can be obtained if we re-write the expression for the IV estimator in (21) as

$$\text{plim}\,\hat{b}_{13} = \frac{\mathbb{E}(g_{it}D_{i,t-5})}{\mathbb{E}(D_{it}D_{i,t-5})} = \frac{a_1 + b_1\lambda\omega(5)}{a_1^2 + \lambda\omega(5)} = \frac{[a_1/\omega(5)] + b_1\lambda}{[a_1^2/\omega(5)] + \lambda}.$$

Consider two extreme cases. First, when $\omega(5) = 1$, i.e., when the 5th order autocorrelation coefficient in error term in the growth equation and the debt equation are the same, the IV estimator collapses to the OLS estimator, i.e.,

$$\text{plim}\,\hat{b}_{13} = \text{plim}\,\hat{b}_1.$$

Second, when $\omega(5)$ becomes large, the IV estimator converges to the "true" value of the underlying parameter, i.e., when $\omega(5) \to \infty$

$$\text{plim}\,\hat{b}_{13} = b_1.$$

Thus, we can see that the crucial condition that reduces the magnitude of the negative bias in the IV estimator is the relative magnitude of the autocorrelation coefficients of the error term in the growth equation, u_{it}, and the error term in the debt equation, v_{it}.

What is the intuition here? Since the IV estimator is the ratio of the reduced form and the first stage partial effects, the numerator in the expression above captures the reduced form effect of lagged debt on contemporary growth, and the denominator captures the first stage effect of lagged debt on contemporary debt. From the numerator we see that the reduced form effect is a weighted average of a_1 and b_1, with relative magnitudes of autocorrelation coefficients and variances functioning as weights.[13] When the magnitude of the autocorrelation coefficient for u_{it} (error in the growth equation) increases relative to v_{it} (error term in the debt equation), then the contribution of a_1 (the reverse causal effect of growth on debt) to the reduced form effect falls, i.e., the confounding effect of the reverse casual relationship is neutralized better by the IV.

In one extreme case, when autocorrelation coefficient of u_{it} (error in the growth equation) is the same as the the autocorrelation coefficient of v_{it} (error term in the debt equation), the IV is useless because the confounding effect of the reverse causal effect is in full force.

[13] To be more precise, the reduced form effect is the numerator divided by the variance of $D_{i,t-5}$. But since this variance is a constant, one can consider it as a normalization factor.

In the other extreme case, when the autocorrelation coefficient of u_{it} (error in the growth equation) is infinitely larger than the the autocorrelation coefficient of v_{it} (error term in the debt equation), the confounding effect of the reverse causal channel is perfectly tamed, and the IV estimator takes us to the "true" value.

To summarize the relationship between the autocorrelation parameters of the data-generating process and the effectiveness of IV as an identification strategy, we observe that when the ratio $\rho_u(5)/\rho_v(5)$ is approximately one, the IV estimate approaches the OLS estimates, i.e., $\text{plim}\,\hat{b}_{13} = \text{plim}\,\hat{b}_1$. When the ratio $\rho_u(5)/\rho_v(5)$ is large, then the IV estimate approaches the true value of b_1, i.e., $\text{plim}\,\hat{b}_{13} = b_1$. Intuition for this result follows from the standard conditions for identification with instrumental variables. There must be some correlation between the excluded exogenous instrument and the endogenous regressor, i.e., $\rho_v(5)$ cannot be too small, but the correlation cannot be so strong that we fail to break the endogenous relationship, i.e., the excluded exogenous instrument must not be as bad as the endogenous regressor.

Continuing with our example of AR(1) errors, the condition for bias reduction becomes: $\phi_u^5 - \phi_v^5 < 0$.

In the lower left-hand panel of Figure C.1, we plot the function, $F(\phi_u, \phi_v) = \phi_u^5 - \phi_v^5$, on the unit square (with the same parameter values as in the upper left figure) and indicate the area on the unit square for which the IV estimator improves on the OLS estimate (and the value of the function is negative).

B.4 Lagged Growth as an Additional Control

In this case we estimate the following model

$$g_{it} = b_{14}D_{it} + b_2 g_{i,t-1} + \varepsilon_{it} \tag{23}$$

instead of (6). Let $\hat{b}_{14}$ be the OLS estimator of b_{14} in (23); then

$$\operatorname{plim} \hat{b}_{14} = \frac{\mathbb{E}\left(g_{i,t-1}^2\right)\mathbb{E}\left(D_{it}g_{it}\right) - \mathbb{E}\left(D_{it}g_{i,t-1}\right)\mathbb{E}\left(g_{it}g_{i,t-1}\right)}{\mathbb{E}\left(g_{i,t-1}^2\right)\mathbb{E}\left(D_{it}^2\right) - \mathbb{E}\left(D_{it}g_{i,t-1}\right)\mathbb{E}\left(D_{it}g_{i,t-1}\right)}. \tag{24}$$

The expression for the probability limit of $\hat{b}_{14}$ can be simplified to the following:

$$\operatorname{plim} \hat{b}_{14} = \frac{(a_1 + b_1\lambda) - [a_1\rho_u(1) + b_1\rho_v(1)\lambda]}{(a_1^2 + \lambda) - \frac{1}{1+b_1^2\lambda}\left[a_1\rho_u(1) + b_1\rho_v(1)\lambda\right]^2}. \tag{25}$$

Proposition 4. *If Assumptions 1 and 2 hold, and if*

$$a_1\left[1 - \rho_u(1)\right] + b_1\left[1 - \rho_v(1)\right] < a_1^2 b_1 + b_1\lambda - \frac{b_1}{1 + b_1^2\lambda}\left[a_1\rho_u(1) + b_1\rho_v(1)\lambda\right]^2 \tag{26}$$

then using $g_{i,t-1}$ as an additional control reduces the bias from below, i.e., $\operatorname{plim} \hat{b}_1 < \operatorname{plim} \hat{b}_{14} < b_1$.

Proof. Since $[a_1\rho_u(1) + b_1\rho_v(1)\lambda] < 0$, and $(1 + b_1^2\lambda) > 0$,

$$\operatorname{plim} \hat{b}_1 = \frac{(a_1 + b_1\lambda)}{(a_1^2 + \lambda)} < \frac{(a_1 + b_1\lambda) - [a_1\rho_u(1) + b_1\rho_v(1)\lambda]}{(a_1^2 + \lambda) - \frac{1}{1+b_1^2\lambda}\left[a_1\rho_u(1) + b_1\rho_v(1)\lambda\right]^2} = \operatorname{plim} \hat{b}_{14}$$

so that the lower bound is always satisfied. On the other hand if

$$a_1\left[1 - \rho_u(1)\right] + b_1\left[1 - \rho_v(1)\right] < a_1^2 b_1 + b_1\lambda - \frac{b_1}{1 + b_1^2\lambda}\left[a_1\rho_u(1) + b_1\rho_v(1)\lambda\right]^2$$

then

$$a_1\left[1 - \rho_u(1)\right] + b_1\left[1 - \rho_v(1)\right] < \left\{a_1^2 + \lambda - \frac{1}{1 + b_1^2\lambda}\left[a_1\rho_u(1) + b_1\rho_v(1)\lambda\right]^2\right\} b_1$$

The expression in the braces on the RHS is positive because

$$\left(a_1^2 + \lambda\right)\left(1 + b_1^2\lambda\right) > \left[a_1\rho_u(1) + b_1\rho_v(1)\lambda\right]^2.$$

Hence, we can divide through by the expression in the braces to get

$$\text{plim}\,\hat{b}_{14} = \frac{(a_1 + b_1\lambda) - \left[a_1\rho_u(1) + b_1\rho_v(1)\lambda\right]}{(a_1^2 + \lambda) - \frac{1}{1+b_1^2\lambda}\left[a_1\rho_u(1) + b_1\rho_v(1)\lambda\right]^2} < b_1$$

This completes the proof. □

To understand the intuition behind the result in proposition 4 let us re-write the expression for the estimator in (25) as

$$\text{plim}\,\hat{b}_{14} = \frac{a_1\left[1 - \rho_u(1)\right] + b_1\lambda\left[1 - \rho_v(1)\right]}{(a_1^2 + \lambda) - \frac{1}{1+b_1^2\lambda}\left[a_1\rho_u(1) + b_1\rho_v(1)\lambda\right]^2} \tag{27}$$

Consider two extreme cases. First, when $\rho_u(1) = \rho_v(1) = 1$, the estimator converges to 0, which gives a positively biased estimate (because the "true" value of the parameter is negative, i.e., $b_1 < 0$). Second, when $\rho_u(1) = \rho_v(1) = 0$, the estimator coincides with the OLS estimator, $\hat{b}_1$. Thus, for the estimator $\hat{b}_{14}$ to reduce the magnitude of the negative bias, the autocorrelation in the error terms must be bounded away from the extreme values of 0 and 1, which is analogous to the case of k-period forwarding that we discussed in proposition 1. The expression in (26) gives the precise domain of values of the combination of autocorrelations, $(\rho_u(1), \rho_v(1))$, on the unit square on the positive part of the 2D plane that is needed to ensure the reduction in the magnitude of the negative bias.

One way to get a better intuitive grasp of the issues here is to use a "partialling out" interpretation of the estimator $\hat{b}_{14}$. Using the Frisch-Waugh-Lovell theorem, $\hat{b}_{14}$ is the OLS estimator from a regression of contemporary growth, g_{it}, on the residual obtained from an auxiliary regression of contemporary debt, D_{it}, on lagged growth, $g_{i,t-1}$. Hence, the numerator of the expression in (25) is the covariance of contemporary growth with the residual, and the denominator is the variance of the residual. Comparing the expression in (25) with the analogous expression in (12), we see that that the numerator increases (becomes less negative) and the denominator increases. Intuitively, including lagged growth as an additional control has two effects (if the conditions stated in proposition 4 are satisfied): first, it soaks up some of the variation in the error term in (6) leading to a lower variance in the resulting error term; and second, it makes the covariance between contemporary growth and the residual of the auxiliary regression less negative than the covariance of contemporary growth and contemporary debt. The net result is that inclusion of a lagged growth term, under these conditions, reduces the magnitude of the negative bias.

Equation 27 gives insight into the conditions of effectiveness of the approach of controlling for lagged growth for recovering the true value of b_1. If $\rho_u(1)$ and $\rho_v(1)$ are both close to 1, the estimate of b_1 based on controlling for lagged growth approaches zero, i.e., $\operatorname{plim} \hat{b}_{14} = 0$, and assuming $b_1 < 0$, average forwarding is positively biased. If $\rho_u(1)$ and $\rho_v(1)$ are both close to zero, the estimate of b_1 based on controlling for lagged growth approaches the OLS estimate, $\operatorname{plim} \hat{b}_{14} = \operatorname{plim} \hat{b}_1$, and controlling for lagged growth is, like OLS, negatively biased.

This result is not surprising because the estimator controlling for lagged growth focuses identification on growth innovations. If there is little innovation, i.e., $\rho_u(1) = 1$, then the approach does not yield useful results. For lower values of $\rho_u(1)$ and $\rho_v(1)$, controlling for lagged growth will recover the true value of b_1.

Continuing with our example of AR(1) errors, the condition for bias reduction, 26, becomes:

$$a_1 \left[1 - \phi_u\right] + b_1 \left[1 - \phi_v\right] - a_1^2 b_1 - b_1 \lambda + \tfrac{b_1}{1+b_1^2\lambda} \left[a_1 \phi_u + b_1 \lambda \phi_v\right]^2 < 0$$

In the upper right-hand panel of Figure C.1, we plot, using the same parameter values as used in the other figures, the function $F(\phi_u, \phi_v) = a_1 \left[1 - \phi_u\right] + b_1 \left[1 - \phi_v\right] - a_1^2 b_1 - b_1 \lambda +$

$\frac{b_1}{1+b_1^2\lambda}\left[a_1\phi_u + b_1\lambda\phi_v\right]^2$ on the unit square, and indicate the combination of (ϕ_u, ϕ_v) for which the value of the function is negative, and hence the estimator based on the inclusion of a Lagged Growth Control improves on the OLS estimate without the Lagged Growth Control.

C Taking Stock of the Bias Reduction Strategies

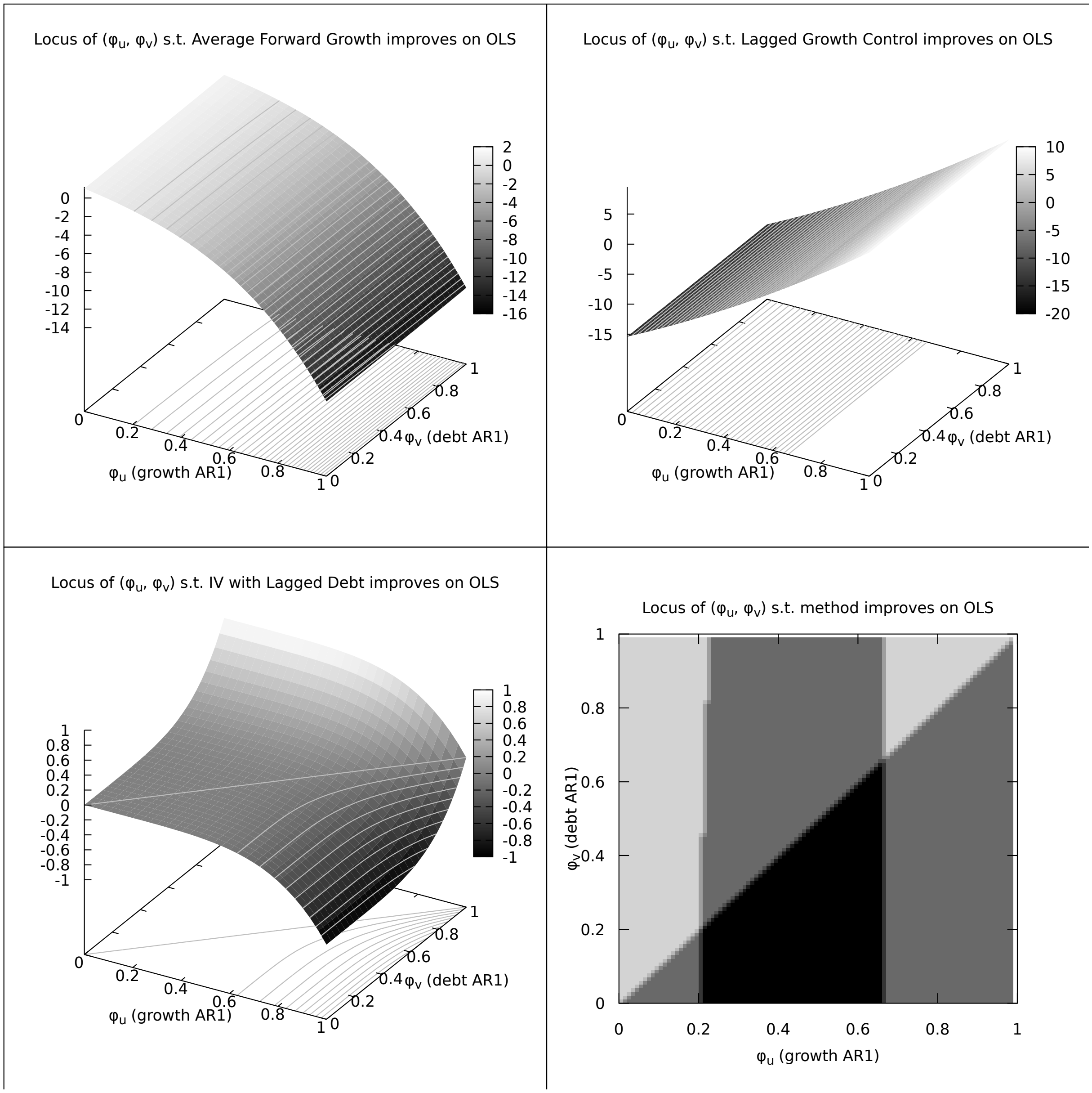
The lower-right panel of Figure C.1 shows the alternative loci for which the three identification approaches improve on the OLS estimates. With our plausible values of $a = -20$, $b = -0.012$, and $\lambda = 16$, the intersection of the loci — the area in which all three of the methods, Lagged Growth Control, Average Forward Growth, and Instrumental Variables, yield improvements over the OLS estimate (which is represented by the dark shaded trapezoid-like region) — occupies a substantial share of the unit square.

But the implication of the bias-reduction analysis is stronger. As we note above, the Lagged Growth Control and the Average Forward Growth approaches focus identification on different parts of the data-generating process. Controlling for lagged growth focuses identification of the effect of debt on innovations in growth, i.e., persistent growth is controlled for via the inclusion of the lag. Average Forward Growth focuses identification on the persistent portion of growth, i.e., the accumulation of growth over a five-year period. These alternative focuses imply different responses of the estimator to alternative values of (ϕ_u, ϕ_v).

The Lagged Growth Control and Average Forward Growth estimators have the convenient property that they reduce bias (or are biased towards zero) under opposite assumptions about ϕ_u. As the upper-left panel of Figure C.1 illustrates, Average Forward Growth, focused on the persistent component, performs better when ϕ_u is relatively high and will be biased toward zero only if ϕ_u is relatively low. As the upper-right panel of Figure C.1 shows, the Lagged Growth Control, focused on innovations, performs better when ϕ_u is relatively low and will be biased toward zero only if ϕ_u is relatively high.

The combination means that it cannot be that both of them are biased towards zero (under the data-generating process). If both reduce the magnitude of the estimate vis-à-vis the baseline OLS estimate—which is the case empirically—then the reduced magnitude constitutes an improvement on the baseline OLS, and cannot a result of specific conditions on (ϕ_u, ϕ_v) that bias the estimate toward zero. That is, the similarity of the estimates for the Lagged Growth Control and the Average Forward Growth specification in the empirical analysis imply that the likely values of (ϕ_u, ϕ_v) are in the intersection of the zones where each strategy reduces bias.

Figure C.1: *Conditions for Bias Reduction*



Notes. The panels illustrate the effect of each of three bias reduction strategies and, in the lower-right panel, the intersection of the three approaches. For Average Forward Growth (the upper-left panel), the shaded locus of improvement (ϕ_u, ϕ_v) is derived from Equation 20. For Lagged Growth Control (the upper-right panel), the shaded locus of improvement (ϕ_u, ϕ_v) is derived from Equation 26. For Instrumental Variables (the lower-left panel), the shaded locus of improvement (ϕ_u, ϕ_v) is derived from Equation 22. These figures were produced using the following values: $a = -20, b = -0.012, \lambda = 16$, and under the assumption that the error terms in the growth and debt equations follow AR(1) processes with coefficients ϕ_u and ϕ_v respectively.

Appendix D Data Appendix

In this appendix, we describe our various data sources in greater detail. A word about the two datasets derived from Reinhart and Rogoff (2010) is in order. Reinhart and Rogoff published related public-use data on their website, but the actual data used in Reinhart and Rogoff (2010) were made available by Reinhart and Rogoff to the authors of Herndon et al. (2014) in spreadsheet format. Herndon et al. released a data and code package based on these data from their website, and this release forms the basis of our analysis. This dataset covers the time period 1946–2009 and includes the 20 countries indicated in column 2 of Table 1. Following the authors' recommendation in Reinhart and Rogoff (2013), we use a subset of these data that limits the data to 1955–2003 and refer to this dataset as RR1955. In the data from Reinhart and Rogoff, public debt is measured as gross central government debt as a percentage of GDP, and growth is measured as the annual growth rate of real GDP (not per capita).

The dataset for Cecchetti et al. (2011) was downloaded from the website of the Bank of International Settlements.[14] This data set, which we call CMZ, covers the 18 countries indicated in column 2 of Table 1 for the period 1980–2009. As explained in Cecchetti et al. (2011), the data come either from the OECD website or from national sources. In this dataset, public debt is measured as gross liabilities of general government valued at market prices on a non-consolidated basis (as a percentage of GDP), and growth is measured as the annual growth rate of per capita real GDP. Other variables, e.g., school enrollment and the level of real GDP, were included on separate spreadsheets.

We reconstructed the dataset for Checherita-Westphal and Rother (2012) by downloading data from the Annual macro-economic database (AMECO) website of the European Commission's Directorate General for Economic and Financial Affairs (DG ECFIN) for the following

[14]See http://www.bis.org/publ/work352.htm.

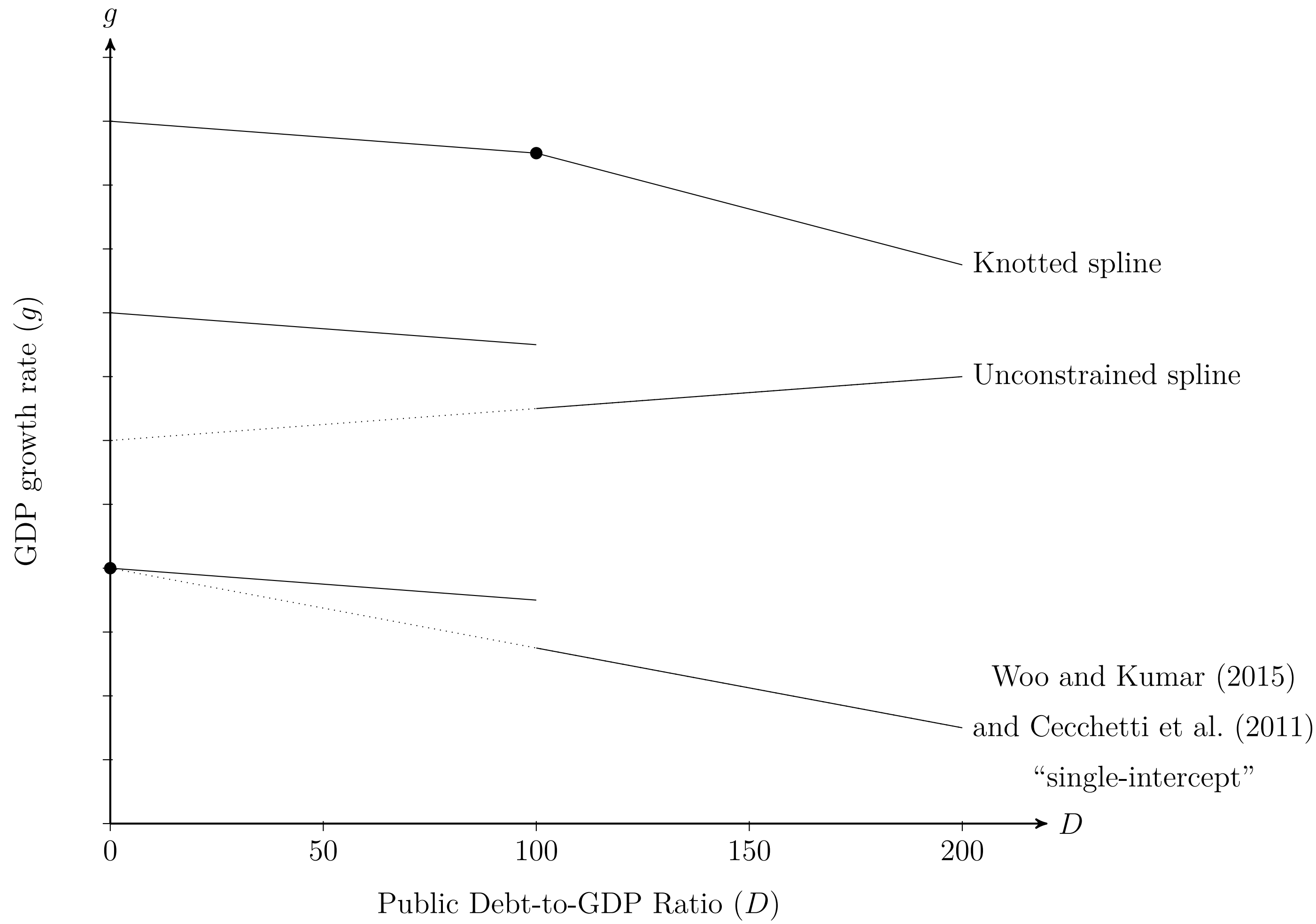
variables: (1) per capita real GDP (gross domestic product at 2010 reference levels per head of population), (2) gross public debt as a percentage of GDP (i.e., general government consolidated gross debt as a percentage of GDP at current prices).[15] This data set, which we call CWR, runs over the period 1970–2008 and includes the 12 countries indicated in column 3 of Table 1. In CWR, public debt is measured as general government consolidated gross debt, and growth is measured as the annual growth rate of per capita real GDP.

We also attempted to reconstruct the advanced economies dataset used by Woo and Kumar (2015). Although Woo and Kumar (2015) cites the IMF World Economic Outlook (WEO) data, the version that we found on the IMF website includes the key macroeconomic and fiscal variables for all countries but all data are limited to 1980–present, while Woo and Kumar (2015) presents results for 1970–present. We reconstructed the data, which we refer to as WK, by merging the WEO data (WEOOct2015all, the set available from the IMF website in October 2015) with data from the Penn World Tables version 7.0 (Heston et al., 2011) and data on fiscal variables for all countries from 1800–present from the IMF Public Finances in Modern History Fiscal Prudence and Profligacy Database (Mauro et al., 2013).[16]

[15] Data were downloaded on 4/14/2016 from http://ec.europa.eu/economy_finance/ameco/user/serie/SelectSerie.cfm.

[16] In no case were the data archived for replication by the journals or authors. Requests for replication data to the corresponding authors for Checherita-Westphal and Rother (2012), Woo and Kumar (2015) did not receive a response. Access to the data for Reinhart and Rogoff (2010) is discussed in Herndon et al. (2014).

Figure D.1: *Schematic of Spline Specifications*



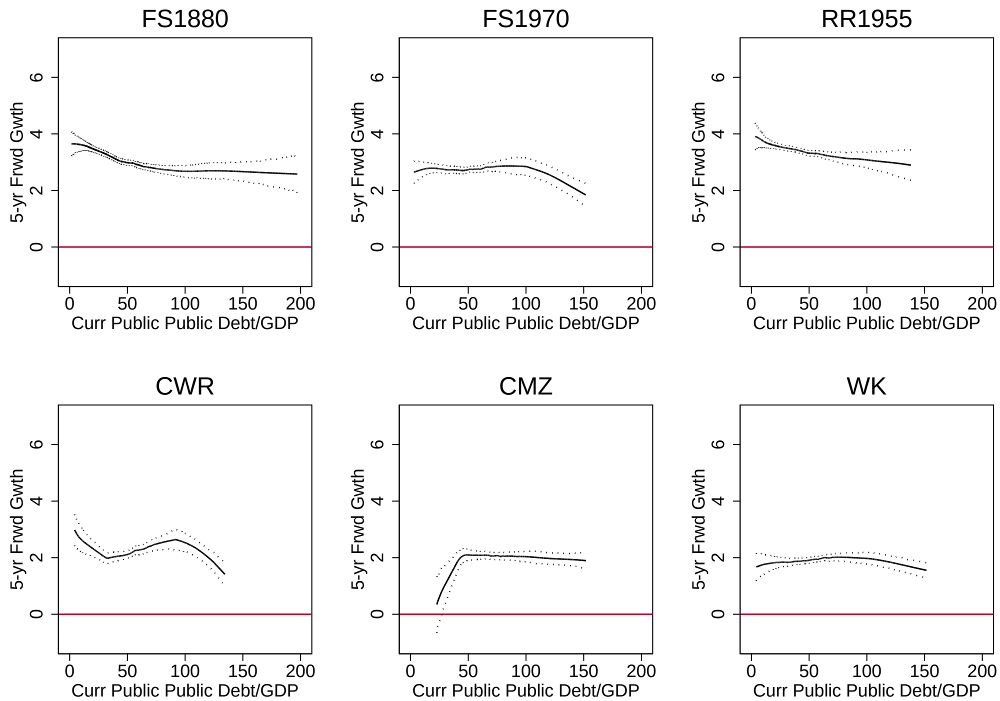
Notes. The diagram illustrates two commonly used spline specifications and the partially constrained spline specification used in Woo and Kumar (2015) and Cecchetti et al. (2011). In the top specification, the spline is knotted at the break point to permit a continuous function with a change in slope. In the middle specification, both a discontinuity and a change in slope are permitted. In the third specification, the two segments are constrained a single common intercept with the vertical axis.

Table D.7: *First-Stage Results for IV Specifications*

	Current Growth			5 Year Future Growth		
	(1)	(2)	(3)	(4)	(5)	(6)
FS1880						
L5.Public Debt/GDP	0.881***	0.874***	0.837***	0.877***	0.870***	0.829***
	(0.020)	(0.020)	(0.030)	(0.020)	(0.020)	(0.027)
Observations	2362	2344	2362	2226	2208	2226
F-Stat	1962	1864	797	2019	1855	956
RR1955						
L5.Public Debt/GDP	0.885***	0.877***	0.865***	0.886***	0.877***	0.865***
	(0.038)	(0.039)	(0.048)	(0.038)	(0.039)	(0.048)
Observations	824	822	824	825	822	825
F-Stat	542	501	326	543	501	326
CWR						
L5.Public Debt/GDP	0.983***	0.984***	0.735***	0.983***	0.984***	0.735***
	(0.046)	(0.022)	(0.185)	(0.046)	(0.022)	(0.185)
Observations	275	275	275	275	275	275
F-Stat	456	1969	16	456	1969	16
CMZ						
L5.Public Debt/GDP	0.893***	0.899***	0.260*	0.893***	0.899***	0.260*
	(0.070)	(0.059)	(0.155)	(0.070)	(0.059)	(0.155)
Observations	252	252	252	252	252	252
F-Stat	165	233	3	165	233	3
WK						
L5.Public Debt/GDP	1.000***	1.002***	0.574***	1.000***	1.002***	0.574***
	(0.057)	(0.058)	(0.140)	(0.057)	(0.058)	(0.140)
Observations	432	432	432	432	432	432
F-Stat	303	299	17	303	299	17
Year FE	Y	Y	Y	Y	Y	Y
Country FE			Y			Y

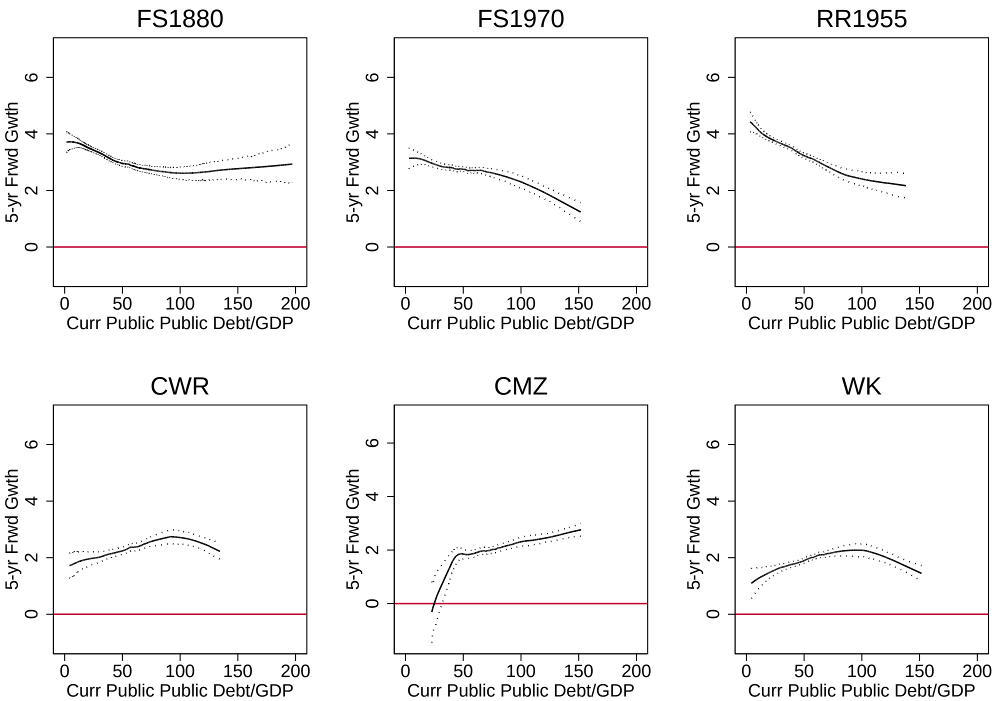
Notes. Future 5-year average growth in real GDP explained by the public debt-to-GDP ratio. Robust standard errors,clustered by country, in parentheses below parameter estimates. Significance levels are as follows: * p<0.1, ** p<0.05, *** p<0.01. For references to the datasets see Section 2.3.

Figure D.2: *Public Debt-Future Growth Partial Linear Regression Controlling for Lagged Growth*



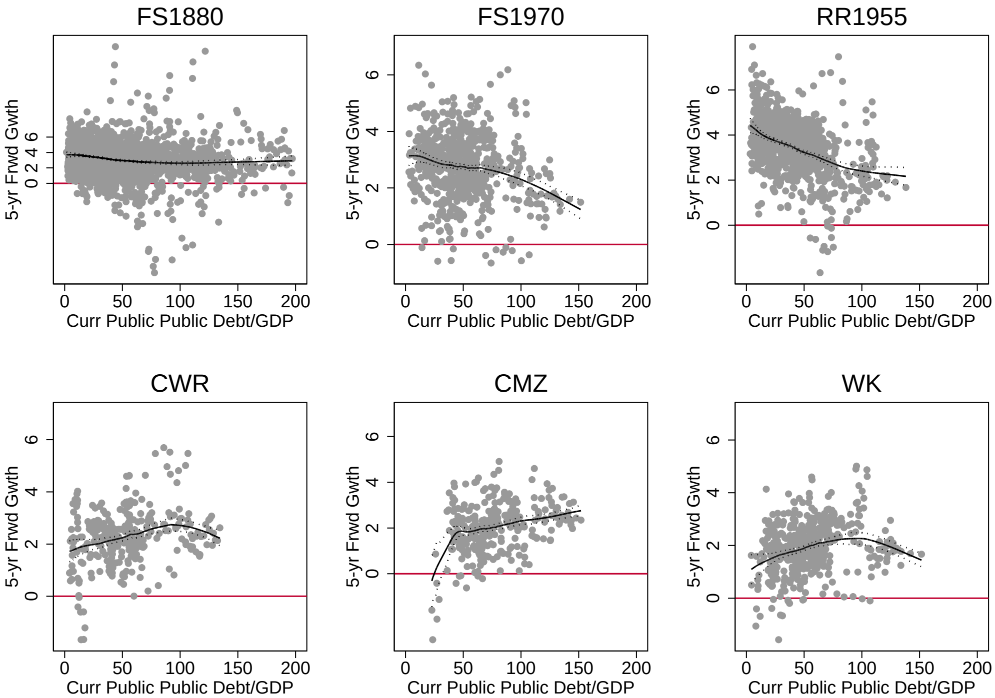
Notes. Partial Linear Regression Residual Lowess plots for the relationship between public debt and five-year future average real GDP growth controlling for one-year lagged real GDP growth and including year dummies. For references to the datasets see Section 2.3. Bootstrapped confidence intervals based on 250 repetitions.

Figure D.3: *Public Debt-Future Growth Partial Linear Regression with Country Fixed Effects*



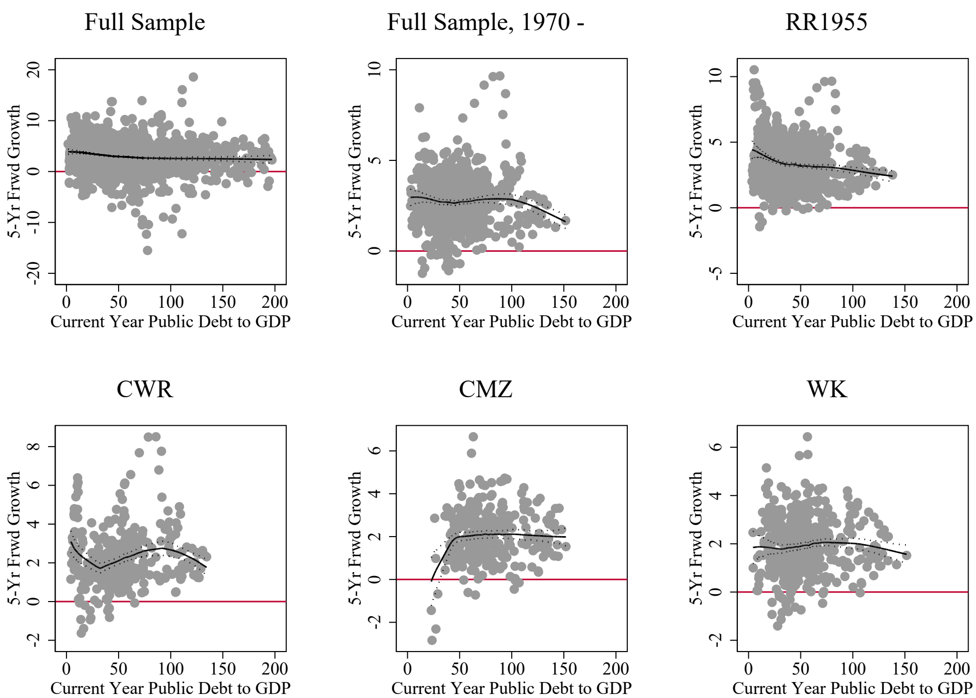
Notes. Partial Linear Regression Residual Lowess plots for the relationship between public debt and five-year future average real GDP growth including country and year dummies. For references to the datasets see Section 2.3. Bootstrapped confidence intervals based on 250 repetitions.

Figure D.4: *Public Debt-Future Growth Partial Linear Regression with Country Fixed Effects with Scatterplot*



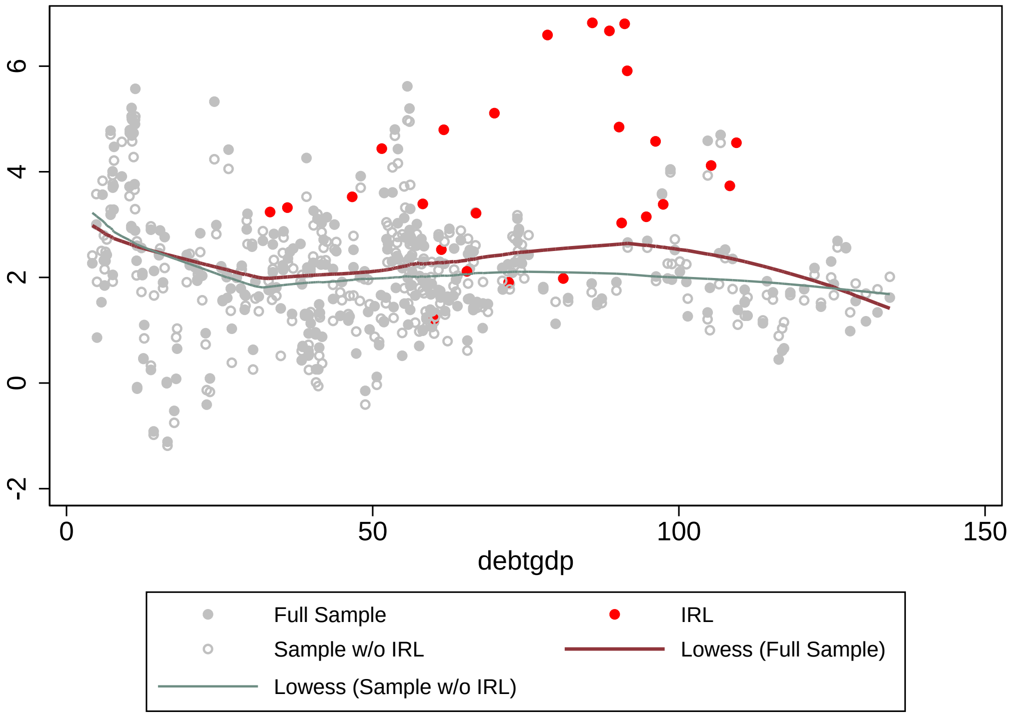
Notes. Partial Linear Regression Residual Lowess plots for the relationship between public debt and five-year future average real GDP growth including country and year dummies. For references to the datasets see Section 2.3. Bootstrapped confidence intervals based on 250 repetitions.

Figure D.5: *Public Debt-Growth Lowess with Scatterplot*



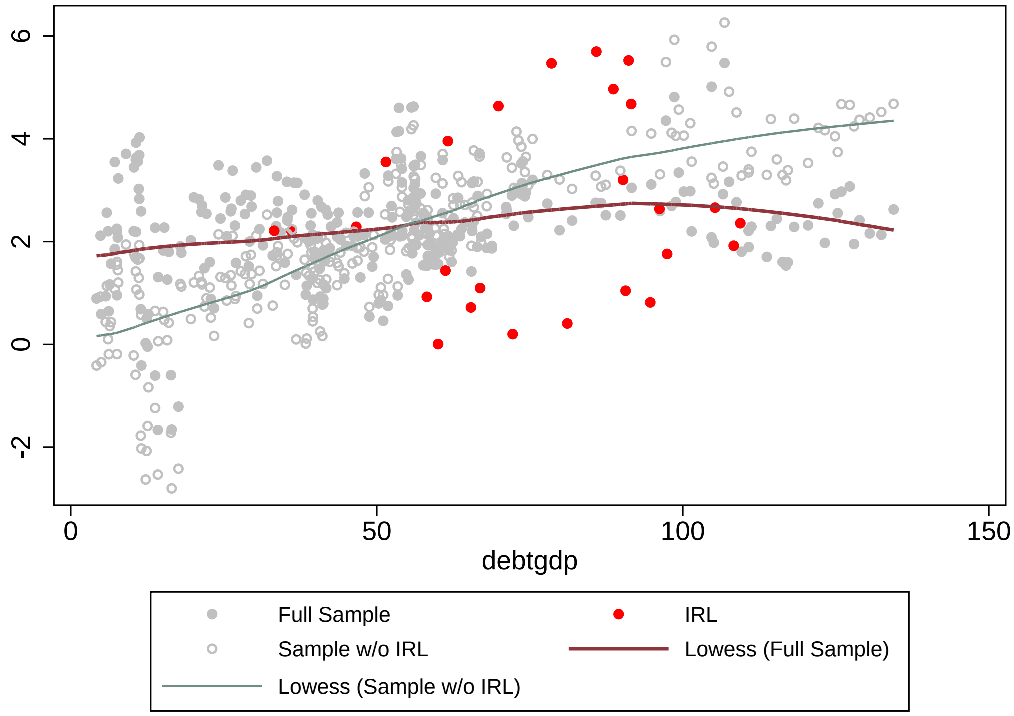
Notes. Lowess plots for the bivariate relationship between public debt and real GDP growth. For references to the datasets see Section 2.3. In each row, the relationship between contemporary public debt and average real GDP growth for the preceding five years is show in the left panel, between contemporary public debt and contemporary real GDP growth in the center panel, between average real GDP growth for the following five years in the right panel. Bootstrapped confidence intervals based on 250 repetitions.

Figure D.6: *Public Debt-Future Growth Partial Linear Regression for CWR with \& without Ireland*



Notes. Partial Linear Regression Residual Lowess plots for the relationship between public debt and five-year future average real GDP growth controlling for one-year lagged real GDP growth and including year dummies. Future For references to the datasets see Section 2.3.

Figure D.7: *Public Debt-Future Growth Partial Linear Regression with Country Fixed Effects for CWR with \& without Ireland*



Notes. Partial Linear Regression Residual Lowess plots for the relationship between public debt and five-year future average real GDP growth including country and year dummies. For references to the datasets see Section 2.3. Bootstrapped confidence intervals based on 250 repetitions.