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Abstract

When does the banking sector profitability exceed the industrial sector profitability? In a benchmark, counterfactual scenario where banks lend to industrial firms only, the industrial profitability exceeds the bank profitability unless the banks' financial intermediation technology is sufficiently advanced *and* the industrial sector's leverage ratio is sufficiently high. In a more realistic scenario where the banks lend not only to the industrial firms and but also to households, which rely on bank loans for housing, vehicle, education, etc., the bank profitability exceeds the industrial profitability when the household sector's total debt services are sufficiently large, regardless of the levels of the banks' financial technology and the industrial sector's leverage ratio. It suggests that the households' debt and interest payments are potentially a important source of the bank profitability exceeding the industrial profitability. That is, the bank capital beat the industrial capital in terms of profitability by exploiting the households' financial conditions that force them to rely on debt for their full reproduction. Lastly, this result is empirically confirmed through the ARDL cointegration analysis with the U.S. quarterly data for 1984–2020.

Keywords: bank profitability, household debt, ARDL cointegration

JEL Classication: G21, G29, G51

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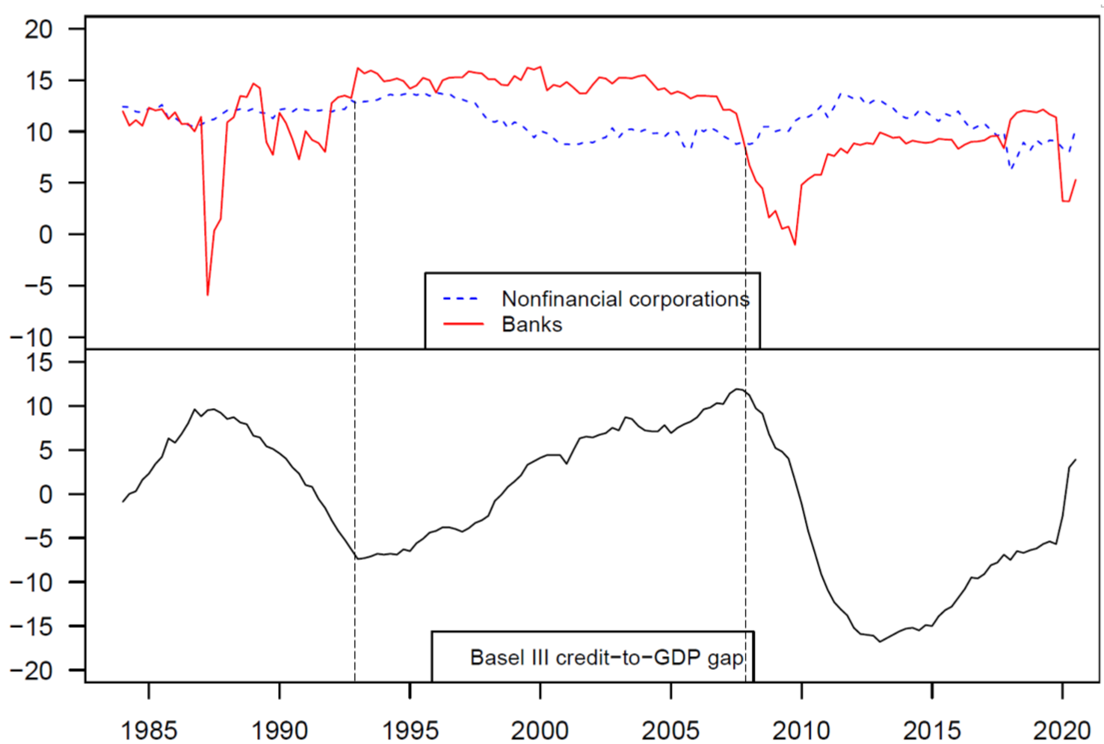
1 Introduction

How does the profitability compare between the banking sector and the industrial sector? According to standard economic theory, real assets determine the productive capacity of the economy and generate net income whereas financial assets, which are claims to the income generated by the real assets, simply define the allocation of income among relevant stakeholders. In this perspective, one would expect a long-term tendency of the industrial sector profitability exceeding the banking sector profitability. However, data show that is not always the case. The top of figure 1 compares the return on equity (ROE, hereafter) of the banks and nonfinancial corporations for the U.S. using the quarterly data for 1984Q1–2020Q3. There is a long period of around 25 years from 1993 up until 2008 financial crisis when the banking sector ROE is consistently greater than the nonfinancial sector ROE. And it was the opposite before and after this 20 year period.

Why do we have to care about the profitability difference between the two sectors? Because it seems to be related to systemic risk. The bottom of figure 1 is the credit to GDP gap, one of the widely used measures of systemic risk suggested in the Basel III framework. It measures how the credit-GDP ratio deviates from its long-run trend. There is a long period of around 25 years from 1993 up until 2008 when the credit-to-GDP gap exhibited a consistently increasing trend. And it is very interesting to see that the start and end points of this period exactly coinincide with the long 25-year period noted above during which the bank profitability consistently exceeded the industrial profitability. Some similar story can be found in the periods before and after the long period. These observations tell us that when the bank profitability is stronger than the nonfinancial firm's profitability it seems to increase systemic risk by generating more credit to the economy relative to the long-run trend, which can also make the system more fragile.

Against this background, in this paper I examine the conditions under which the bank profitability exceeds the industrial profitability theoretically and empirically. First, I present a model where there are two types of capitalists, i.e., capitalists who are equipped with heterogeneous industrial technology to operate a productive business and bankers who are equipped with homogeneous financial intermediation technology to issue debt and loan. All of these capitalists are also endowed with the same amount of wealth. The capitalists with sufficiently advanced industrial technology optimally decide to use their initial wealth to op-

Figure 1: The return on equity of the nonfinancial corporations vs. banks: 1984,Q1–2020,Q3, quarterly



Source: FRED Economics Data, St. Louis Fed

erate the productive business whereas those with less advanced technology optimally decides to use their wealth to purchase bank debt, thereby becoming a financial capitalist; I call the bankers and the financial capitalists combined together financial sector. From this emerges industrial sector, banking sector, and financial sector, and I derive the ROE of each of these three sectors and identify the conditions underlying their rankings.

For this, I consider two different cases. The first is a baseline, counter-factual case where the household sector is simply ignored and hence the banks only lend to the industrial sector, while the second is an extended case where the household sector is explicitly considered. In the latter, it is assumed that debts are incurred only to support consumption for living and reproduction but not financial investment, and it is also assumed that the capitalist households have sufficient amount of wealth and income so that they do not incur debt, which however is not the case for the worker households. Hence, in second case, the worker households as well as the industrial capitalists are the ones who borrow from the banks.

Here are the key results, focusing on the comparison between the industrial and banking sectors. In the baseline case without the household borrowing, on the one hand, the industrial profitability is greater than the bank profitability unless the following two conditions are met simultaneously, i.e., the financial technology being sufficiently advanced *and* the industrial sector's leverage ratio being sufficiently high. In other words, only when both of these two conditions are met will the bank profitability exceed the industrial profitability. In the model, the degree of financial technology is proxied by the interest rate on the bank debt, or the deposit rate, with a lower (higher) rate indicating a more (less) advanced financial technology. Hence, the advanced financial technology allows the banks to raise funds at a low cost, while the high industrial sector's leverage ratio increases the demand for the bank loans thereby raising the loan interest rate. Both of these contribute to increasing the bank profits.

On the other hand, in the second case where the household borrowing is incorporated, the result depends on the worker household sector's total interest payments. That is, as long as the latter is sufficiently high, the bank profitability will be necessarily greater than the industrial profitability regardless of the financial technology or the industrial sector's leverage ratio. By providing the loan services to the household sector as well, the banks are able to expand the sources of revenues thereby enhancing their profitability.

In the empirical part of the paper, I employ the ARDL cointegration analysis to confirm the theoretical relationship between the profitability differential between the banking and industrial sectors and the household interest payments, using the U.S. quarterly data during 1984,Q1–2020,Q3. The econometric result suggests that there is a long-run equilibrium relationship between the two variables.

In all, both the theoretical and empirical results suggest that the potential source of the banking capital profitability exceeding the industrial capital profitability is the households debt and their interest payments. The main implication of the paper is that the banking capital beats the industrial capital in terms of profitability by exploiting the households' financial conditions that require them to rely on debt for their living and reproduction.

The model presented in this paper is an extension of Park (2021). In the latter, there is no banker but only capitalists equipped with industrial technology heterogeneous across the capitalists. Those equipped with more advanced technology optimally choose to use their wealth in operating the productive activity thereby becoming an industrial capitalist, whereas those equipped with less advanced technology optimally chose to lend their wealth to the former thereby becoming a financial capitalist; here, the loan is made directly from the lender to the borrower without financial intermediaries. In this paper, I extend Park (2021)'s model of direct finance to incorporate banking capitalists so that the financial capitalists now invest their wealth in the bank debt rather than directly lending to the industrial capitalists.

Note that in Park (2021)'s model, the capitalists become a financial capitalist not because they have financial technology but because their industrial technology is not strong enough to warrant investing their wealth in the productive activity. Consequently, the yields they achieve from their financial investment, which is their fallback option, is necessarily less than the yields of industrial capitalists who are equipped with strong technology. Hence, it cannot be otherwise but that the financial sector's average profitability is less than that of the industrial sector. In comparison, the current paper introduces to the model the bankers who are equipped with financial technology. And the capitalists equipped with weak industrial technology indirectly lend their wealth to the industrial sector through the banks. This setup allows the possibilities of the banking and financial sectors' profitability exceeding the industrial sector's profitability. And one of the factors that underlies these possibilities is the relative strength between the financial technology and the industrial technology.

In contrast to the current paper's case of dealing with the differentials between the industrial and banking profitability, the literature that examines the banking sector profitability alone is vast. First, there are empirical papers that identify its determinants. Some of the recent papers include, e.g., Chen et al. (2024), Lamothe et al. (2024), Le and Ngo (2020), Saif-Alyousfi (2022), and Menicucci and Paolucci (2016), etc. These papers consider bank-specific factors such as size, capital ratio, liquidity ratio, net interest margin, and non-performing loans, etc. and macroeconomic factors such as economic growth, unemployment ration, inflation, etc. studying how each of these affects the bank profitability.

Another important stream of research worthwhile to note is those that examine profitability, or efficiency, of banking in relation to competition and market power; most of the papers here are based on the structure-conduct-performance (SCP) hypothesis and relative-market-power (RMP) hypothesis or the new empirical industrial organization such as Panzar and Rosse (1987) reduced-form revenue model or the Lerner Index method. For instance, using European data for 1986–1989, Molyneux and Thornton (1992) find support for a positive correlation between market concentration and bank profitability. Using the data for 1999–2008, Mirzaei et al. (2013) observe a positive correlation between market share and bank profitability in advanced economies, but not in emerging economies. Contrarily, using the U.S. annual data for the period of 1976–2006, Goetz (2018) finds that competition supports bank profitability. The results in Yuanita (2019) that uses the Indonesian data for 2000–2015 depend on the measures of competition and hence are more nuanced; in the case of a structural measure such as a concentration ratio, it is found that competition positively affects the bank profitability, whereas in the case of a non-structural measure such as the Lerner Index the impact is negative.

Against the background of the literature briefly discussed thus far, the current paper is, to my knowledge, the first that analyzes the profitability differentials between the banking and industrial sectors. If the hypothesis that the bank profitability exceeding the industrial profitability can intensify systemic risks is proved valid—which is a topic for a separate research—then the differential and its determinants are something the policy makers should pay attention.

The rest of paper is organized as follows. A baseline model without household borrowing is presented in section 2.1, and an extended model with household borrowing is presented in section 3. A cointegration analysis is conducted in section 4 to verify the theoretical results of the paper. Section 5 is a conclusion.

2 Model

2.1 Basic setup

Consider an economy where there is a productive business opportunity that requires inputs of capital $\lambda > 1$ and labor λ/τ, where τ is the capital-labor ratio and labor refers to the number of workers. There are a sufficiently large number of workers so that the labor supply is unconstrained; hence, the wage per worker is fixed at w, which is the minimal level corresponding to the consumption required to maintain the basic, or average, standard of living. Wages are paid out of the revenues from the business and therefore the investment required to initiate the business is λ. The business is available to anyone who can finance it and has technology to operate it. Consider an agent with technology, which can yield net output per capital of y from the business. Then the agent's return from operating the business is, denoting it by r, by defintion,

$$r = y - \frac{w}{\tau} \tag{1}$$

Since y is equivalent to capital productivity, labor productivity is measured as $y\tau$.

In addition to workers, the economy is populated by a number of capitalists and bankers. The bankers are endowed with wealth and financial intermediation technology. The bankers use their wealth as equity to form banks; the combined wealth of all the bankers are normalized to unity. In addition, they also raise funds by issuing debt, an example of which is deposit. The cost of debt is i^D. All the funds are used for lending to the industrial capitalists—yet to be discussed—at the rate of i^K. The bankers' financial intermediation technology consists of risk management and skills related to issuance, monitoring, and collection of loans, etc. all of which combined together allow the banks to raise funds at a competitive rate. In this respect, the level of i^D is considered as reflecting the status of the financial intermediation technology, i.e., a lower (higher) i^D implying a more (less) advanced financial technology.

The capitalists are endowed with wealth and technology for operating the productive business. On one hand, the technology is characterized by y which varies across the capitalists. On the other hand, the wealth endowment, which is normalized to unity, can be used in two different ways; either to finance the initiating the business, in which case the capitalist becomes an industrial capitalist, or to invest in the bank debt, in which case she becomes a financial capitalist as a creditor of a bank; a financial capitalist in the sense that her funds are eventually lent to borrowers via the bank's intermediation functions.[1] Since the productive business requires an initial investment of λ but a capitalist's wealth is only unity, the capitalist who decides to perform the productive activity is required to borrow from a bank by the amount of $\lambda - 1$. Note that λ also indicates the leverage ratio, which is defined as asset over equity.

The bankers are also capitalists. Narrowly defined, they are banking capitalists in the sense that they earn profits by providing banking services using their financial intermediation technology; more broadly, they are also financial capitalist in the sense that their wealth—in the form of bank equity—are eventually lent to borrowers. In contrast to the capitalists, including bankers, workers have neither wealth nor technology of any kind.

In a financial context of lending and borrowing, when comparing the profitability between lenders and borrowers, the return on equity is a proper measure of profitability; it is defined

[1] A direct lending of the financial capitalists' funds to borrowers without going through banks is considered in Park (2021) but not in this paper.

as net profits, considering all the earnings and payments of interest, divided by net worth, or equity. It measures the profitability of one's initial out-of-pocket funds in a circumstance where the person can borrow to scale up the investment beyond the own funds. The return on equity can be useful in comparing the performances of different ways of employing the same amount of initial endowments. For example, consider a capitalist equipped with technology that yields y; when she operates the productive business, the return on equity, denoted by r^e, is

$$r^e \equiv \lambda \left(y - \frac{w}{\tau} \right) - (\lambda - 1) i^K \tag{2}$$

The decision the capitalist has to make is whether to employ her wealth in the productive activity and earn r^e or to employ it in investing in the bank debt and earn i^D, which is her fallback option. It would be optimal for the y-capitalist to choose the former option when $r^e > i^D$ and the latter when $r^e < i^D$. From these we can derive the threshold level of y as follows, at which the capitalist is indifferent between the two options.

$$\widehat{y} = \left(1 - \frac{1}{\lambda} \right) i^K + \frac{w}{\tau} - \frac{i^D}{\lambda} \tag{3}$$

In the basic setup thus far, the optimal decision the capitalists will make is depicted in the following lemma.

Lemma 1 *A capitalist, equipped with wealth of unity and y-technology, facing a set of interest rates, i^K and i^D, makes the following optimal decision:*

(i) If $y > \widehat{y}$, the capitalist invests her endowment in the productive activity along with the leverage at the ratio of λ, thus becoming an industrial capitalist, and earns r^e.

(ii) If $y < \widehat{y}$, the capitalist invests her endowment in the bank debt and becomes a financial capitalist, and earns i^D.

Lemma 1 demonstrates the division of the capitalist class into industrial and financial capitalists driven by technology differentials. It suggests that in an economy where the capitalists are equipped with heterogeneous productive technology and a homogeneous amount of wealth, which is less than what is required to conduct a capitalist production, those with more advanced technology will engage in the productive activity and become industrial capitalists, whereas those with less advanced technology will use their wealth for financial investment and become financial capitalists.

This way of the capitalist class being divided into the industrial and the financial via technology differentials results in the difference in the average profitability of each of the two capitalist groups. Note that all the industrial capitalists are better-off than all the financial capitalist, which is obvious since the former earn $r^e > i^D$ while the latter earn i^D. Accordingly, if the average return of each sector is compared, it is evident that the industrial sector will dominate the financial sector. The main driver of this result is that the financial capitalists had to choose the fallback option, i^D, due to their weaker productive technology. On the other hand, if the financial sector is expanded to include the banking capitalists the comparison of the capital profitability becomes indeterminate as will be discussed more thoroughly below.

Since the threshold $\widehat{y}$ divides the capitalists into the two groups and accordingly decides the size of each, the determination of $\widehat{y}$ affects the crucial factors of the model such as the demand and supply of bank loans and the size of the total productive activity in the economy, etc. Hence, understanding equation (3) is important in grasping the underlying dynamics of the model. Consider, for instance, an increase in λ which leads to an increase in $\widehat{y}$. For all industrial capitalists to be able to operate with more bank loans, the supply of funds for the banks should rise; this is achieved when $\widehat{y}$ increases in which case the number of financial capitalists who provide funds to the banks is greater while the number of industrial capitalists who demand the greater amount of bank loan is less. How $\widehat{y}$ relates to the other parameters in equation (3) can be understood in similar ways.

For another example, consider an increase in the banks' cost of funding i^D which leads to, according to equation (3), an increase in $\widehat{y}$. Since i^D is an opportunity cost of the productive activity, its increase implies that due to the now greater cost, an industrial capitalist's technology has to be more advanced to make the productive activity viable. Accordingly, given the distribution of y, those industrial capitalists at the margin will now find it optimal to switch to become a financial capitalist and earn i^D; hence, an increase in $\widehat{y}$.

2.2 The market for bank loans

Let us consider the demand and supply of bank loans to determined the equilibrium loan interest rate. As each industrial capitalist's demand for bank loan is $(\lambda - 1)$, the total demand is the sum of it across all the industrial capitalists. Since the capitalists equipped with technology yielding $y > \widehat{y}$ become an industrial capitalist, the total demand for bank loans can be expressed as follows.

$$\int_{\widehat{y}}^{\infty} (\lambda - 1)\mathrm{d}F(y). \tag{4}$$

As for the supply of bank loan, since the banks use all the funds from debt and equity in making loans, which are their only assets, it will be useful to start with considering debt and equity of the banks. On one hand, the total equity of the banking sector is simply unity. On the other hand, all the capitalists with technology of $y < \widehat{y}$ use their wealth of unity in purchasing the bank debt. Hence the total bank debt, denoted by D, is the sum of them across all such capitalists.

$$D \equiv \int_{0}^{\widehat{y}} \mathrm{d}F(y) \tag{5}$$

Due to the balance sheet identity, the total supply of bank loans is $1 + D$, and hence,

$$1 + \int_{0}^{\widehat{y}} \mathrm{d}F(y) \tag{6}$$

To obtain the equilibrium analytically, the following assumption on the distribution of y is adopted.

Assumption 1 *y is uniformly distributed over* $[0, \overline{y}]$.

Using the expression for $\widehat{y}$ in equation (3) along with assumption 1, we can solve the integrals and confirm that the demand curve is downward-sloping and the supply curve is upward-sloping. More importantly, we can explicitly obtain the equilibrium rate of interest for bank loan. Denoting the latter by i^{K*}, with the superscript star indicating the variables in equilibrium, we obtain it from the equality of demand and supply as follows.

$$i^{K*} = \frac{1}{\lambda - 1}\left[(\lambda - 2)\overline{y} - \lambda\frac{w}{\tau} - i^{D}\right]. \tag{7}$$

Substituting i^{K*} into equation (3) yields the equilibrium threshold of y as

$$\widehat{y}^{*} = \left(1 - \frac{2}{\lambda}\right)\overline{y} \tag{8}$$

There are a couple of trivial or degenerate cases that could possible emerge depending on the parameter ranges. One is $\widehat{y}^{*} \leq 0$ which implies that all the capitalists become industrial capitalists and hence the demand for bank debt is zero, yielding a trivial case of the banks having zero leverage. The other is a degenerate case where the total profits produced by the aggregate productive assets in the industrial sector being zero or negative. To rule these out and confine the analysis below to non-trivial and non-degenerate cases, the following assumption on the parameter space is adopted.

Assumption 2 *(i)* $\lambda > 2$, *(ii)* $\overline{y} > \frac{2w}{\tau}$.

Part (i) of assumption 2 ensures $\widehat{y}^* > 0$ which is easy to verify; under the condition of part (i), part (ii) ensures that the total profits produced by the aggregate productive assets in the industrial sector are positive which is discussed more in detail below.

Lemma 2 is a comparative static analysis of how i^{K*} responds to a change in the parameters.

Lemma 2 *(i)* $\frac{\partial i^{K*}}{\partial \overline{y}} > 0$, *(ii)* $\frac{\partial i^{K*}}{\partial \tau} > 0$, *(iii)* $\frac{\partial i^{K*}}{\partial w} < 0$, *(iv)* $\frac{\partial i^{K*}}{\partial \lambda} > 0$, *(v)* $\frac{\partial i^{K*}}{\partial i^{D}} < 0$.

All the results in lemma 2 are as expected and intuitive. For instance, consider a change in the capital-labor ratio τ in part (ii). τ is a technical feature of the productive business available to all the capitalists and therefore an increase in τ implies an increase in the labor productivity $y\tau$ and, as shown in equation (1), an increase in the return r of the productive business for all. This allows those financial capitalists at the margin to optimally make a transition to the industrial sector. As a result, the number of financial capitalists falls and the number of industrial capitalists rises as reflected in the decrease in the threshold $\widehat{y}$ caused by a rise in τ; see equation (3). From this immediately follows an increase in the demand and a decrease in the supply in the bank loan market and eventually a rise in the loan interest rate, which is what part (ii) suggests.

For another example, consider part (v) on the effect of i^D, which is the opportunity cost of the productive activity. An increase in i^D has an effect of raising the hurdle of becoming an industrial capitalist and therefore those industrial capitalists at the margin will now find it optimal to shift to the financial sector. That is, the increase in i^D raises the threshold $\widehat{y}$ thereby decreasing the number of industrial capitalists and increasing the number of financial capitalists and eventually lowering the loan interest rate. Interestingly, part (v) suggests that when the bank debt rate rises the net interest rate spread is squeezed whereas when the bank debt rate falls the net interest rate spread is widened.

Lemma 3 is a comparative static analysis of how $\widehat{y}^*$ responds to a change in the parameters.

Lemma 3 *(i)* $\frac{\partial \widehat{y}^*}{\partial \overline{y}} > 0$, *(ii)* $\frac{\partial \widehat{y}^*}{\partial \lambda} > 0$.

For the underlying mechanism that drives these results, the earlier discussion on shifts in $\widehat{y}$ applies the same. The difference to note is that the comparative static analysis results for $\widehat{y}^*$ incorporates the equilibrium effects. As will be discussed more in detail below, the equilibrium effect of a change in a parameter works through first moving $\widehat{y}^*$ thereby changing the equilibrium number of the industrial and financial capitalists. In this respect, lemma 3 will be central to the comparative statistic analyses below.

2.3 The profitability of capital

We now turn to the main issue of the profitability of capital. Different types of capital have emerged in the model. The first is the industrial capitalists' own funds which, along with bank loans, finance the productive business. The second is the financial capitalists' own funds which are invested in banks, or, which are lent to borrowers through banks. The third is the bankers' own funds which, along with the debts, finance the loans. The first is industrial capital, the second and the third combined together is financial capital, and the third can also be more narrowly defined as banking capital.

Before examining the profitability of each of these, let us first examine the profitability of the assets of the productive business at the aggregate level—which can also be called the total social capital—regardless of how it is financed. A proper measure of it would be the standard rate of profit, i.e., the aggregate profits yielded by the aggregate productive capital divided by the latter. Denoting it by R, we have

$$R \equiv \frac{\int_{\widehat{y}}^{\infty} \lambda \left(y - \frac{w}{\tau}\right) \mathrm{d}F(y)}{\int_{\widehat{y}}^{\infty} \lambda \mathrm{d}F(y)}, \tag{9}$$

which reflects the fact that R is nothing but the average of r's of all the capitalists with technology of $y > \widehat{y}$. Solving the integrals and substituting the equilibrium interest rate of bank loan obtained in equation (7) yields the equilibrium profit rate of the total social capital as

$$R^* = \left(1 - \frac{1}{\lambda}\right) \overline{y} - \frac{w}{\tau}. \tag{10}$$

To avoid a degenerate case of the total profits from the aggregate productive business being zero or below, only the parameter spaces that yield $R^* > 0$ are considered and this is ensured by part (ii) of assumption 2 under the condition of part (i).

Lemma 2 is a comparative static analysis of R^*.

Lemma 4 *(i)* $\frac{\partial R^*}{\partial \overline{y}} > 0$, *(ii)* $\frac{\partial R^*}{\partial \tau} > 0$, *(iii)* $\frac{\partial R^*}{\partial w} < 0$, *(iv)* $\frac{\partial R^*}{\partial \lambda} > 0$.

The results of lemma 4 are all intuitive and as expected. For instance, in part (iv) an increase in the industrial sector's borrowing λ leads to, as shown in part (ii) of lemma 3, an increase in the threshold $\widehat{y}^*$, making the industrial sector a smaller set of the capitalists with higher y's, i.e., more advanced technology, and hence higher r's. Since the return of the total productive assets is the average of r's of the individual industrial capitalists' productive assets, the end result is its increase in equilibrium.

In comparison to the aggregate productive assets, in measuring and comparing the profitability of industrial and financial sectors that are connected to each other through lending and borrowing, a proper measure is, as mentioned earlier, the return on equity. First, the return on equity of total industrial capital, denoted by R^K, is measured as follows.

$$R^K \equiv \frac{\int_{\widehat{y}}^{\infty} [\lambda \left(y - \frac{w}{\tau}\right) - (\lambda - 1) i^K] \mathrm{d}F(y)}{\int_{\widehat{y}}^{\infty} \mathrm{d}F(y)}. \tag{11}$$

The numerator is the total profits net of the interest payments and the numerator is the total equity of the industrial sector.

Solving the integrals under assumption (1) and using the equilibrium interest rate of bank loan, the equilibrium return on equity of the industrial sector is obtained as

$$R^{K*} = \overline{y} + i^D. \tag{12}$$

It is notable that the equilibrium return on equity of the industrial sector is the sum of the maximal return of the productive technology and the return on the bank debt. From this easily follows lemma 5 on the comparative static analysis of R^{K*}.

Lemma 5 *(i)* $\frac{\partial R^{K*}}{\partial \overline{y}} > 0$, *(ii)* $\frac{\partial R^{K*}}{\partial i^D} > 0$.

While part (i) is intuitive, part (ii) needs an explanation. An increase in i^D, as shown in lemma 2, first leads to a decrease in the loan interest rate i^{K*}. Since the interest paymyent is a minus factor in R^K by definition, the result is an increase of the latter in equilibrium, which is what part (ii) suggests. One may be curious why R^{K*} is not a function of the leverage ratio λ as it seems to be in contrast to the standard DuPont identity, which shows that the leverage ratio has an amplifying effect on the return on equity. But confirm that the amplifying affect of leverage does appear in the definition of R^K in equation (11). However, the increase in λ also has a countervailing effect of raising the loan interest rate in equilibrium as shown in part (ii) of lemma 2. R^{K*} incorporates all the equilibrium effects and hence takes both effects of λ into account. And equation (12) suggests they cancel out each other so that λ is neutral to R^{K*} .

Next, regarding the profitability of the banking sector, let us first consider the bank profit. By definition, it is the revenues from loans minus the funding costs, which are the interest payments on debts. Accordingly, the total profits of the banking sector can be expressed as $i^K(1+D) - i^D D$, where D is the total bank debt as defined in equation (5) and recalling that the total bank equity is unity. In all, the return on equity of the total banking capital, denoted by R^B, is measured as follows.

$$R^B \equiv i^K(1+D) - i^D D. \tag{13}$$

Note that, by construct, the total bank profits and the return on equity of the total banking capital are identical with each other since the total bank equity is one. Equation (13) can be solved to yield the equilibrium return on equity of the banking sector as follows.

$$R^{B*} = 2\left[\left(\frac{\lambda-2}{\lambda}\right)\overline{y} - \frac{w}{\tau}\right] - i^D. \tag{14}$$

Lemma 6 is the comparative static analysis of R^{B*}.

Lemma 6 *(i)* $\frac{\partial R^{B*}}{\partial \overline{y}} > 0$, *(ii)* $\frac{\partial R^{B*}}{\partial \tau} > 0$, *(iii)* $\frac{\partial R^{B*}}{\partial w} < 0$, *(iv)* $\frac{\partial R^{B*}}{\partial \lambda} > 0$, *(v)* $\frac{\partial R^{B*}}{\partial i^D} < 0$

which are all intuitive and as expected. All of the five parameters affect R^{B*} through imparting an impact on the loan interest rate, which is the source of bank profits. Hence, all the sign values of the results are same as those for the comparative statistic analysis of i^{K*} in lemma 2. There are other channels, as well. For instance, as i^D is the funding cost for the banks, it directly has a negative impact on R^{B*}.

Lastly, we examine the financial sector that combines the bankers and their investors. Their total profits are the sum of the profits of the banks and the earnings, $i^D D$, of the bank investors. The return on equity of the total financial capital, denoted by R^F, is obtained by dividing the total profits by the combined own funds of the bankers and their investors. Since the bankers' funds are the total bank equity—which is unity—and the bank investors' funds are used in purchasing the bank debt, R^F is expressed as follows.

$$R^F \equiv \frac{i^K(1+D)}{1+D} = i^K. \tag{15}$$

It turns out that the return on equity of the financial sector is equivalent to the loan interest rate.

Hence, its equilibrium value will also be the same as the equilibrium loan interest rate.

$$R^{F*} = \frac{1}{\lambda - 1}\left[(\lambda - 2)\overline{y} - \lambda\frac{w}{\tau} - i^D\right] = i^{K*} \tag{16}$$

The main function of the financial sector in the model is to lend the own funds—of the bankers and the bank investors—to borrowers. Accordingly, it is obvious that the return of the financial sector is equivalent to the lending rate as equations (15) and (16) indicate. The comparative static analysis of R^{F*} will be the same as that of i^{K*} as follow.

Lemma 7 *(i)* $\frac{\partial R^{F*}}{\partial \overline{y}} > 0$, *(ii)* $\frac{\partial R^{F*}}{\partial \tau} > 0$, *(iii)* $\frac{\partial R^{F*}}{\partial w} < 0$, *(iv)* $\frac{\partial R^{F*}}{\partial \lambda} > 0$, *(v)* $\frac{\partial R^{F*}}{\partial i^D} < 0$.

We are now ready to tackle the central issue of comparing the profitability of the three different types of capital. For a better understanding of the results, some preliminary discussions will be helpful on two crucial aspects of capital profitability that are fundamental in the model. First, the basis of an entitlement to profits is a possession of wealth. The latter is invested either productively or financially and earn returns accordingly. In this relation, the model thus far has presented all the capitalists including bankers as having wealth whereas the workers as the agents who do not own wealth.

Second, the differences in profitability among the different types of capitalists lie in the differences in their technology. The industrial capitalists have productive technology and the banking capitalists have financial intermediation technology. What underlies the connection between technology and capital profitability is that technology makes its owner eligible for taking leverage, which in turn enhances profitability. For instance, consider a capitalist with $y < \widehat{y}$ who optimally chooses her fallback option i^D instead of becoming an industrial capitalist. The reason for this choice is that she are not able to borrow to finance the productive business since her weak technology does not make the borrowing at the ongoing rate profitable. On the other hand, as mentioned earlier, the financial intermediation technology enables the banks to raise funds at a competitive cost. If the financial technology is weak or not sufficiently advanced so that the funding cost for the banks is as high as the lending interest rate, the banks will not issue debt and hence their leverage will be zero. Accordingly, the bank profitability will tend to be depressed.

In the context of this connection between technology and capital profitability, proposition

1 reports the results on how the different types of capital compare with each other in terms of profitability.

Proposition 1 *In the model economy under the parameter spaces described in assumptions 1 and 2 the following relations hold among the industrial capital, the banking capital, and the financial capital.*

(i) $R^{K*} > R^{F*}$.

(ii) *In case* $i^D > \widehat{i}^1$, $R^{K*} > R^{B*}$ *and in case* $i^D < \widehat{i}^1$, $\lambda \lesseqgtr \widehat{\lambda}^1 \Longleftrightarrow R^{K*} \gtreqless R^{B*}$.

(iii) *In case* $i^D > \widehat{i}^2$, $R^{B*} < R^{F*}$ *and in case* $i^D < \widehat{i}^2$, $\lambda \gtreqless \widehat{\lambda}^2 \Longleftrightarrow R^{B*} \gtreqless R^{F*}$

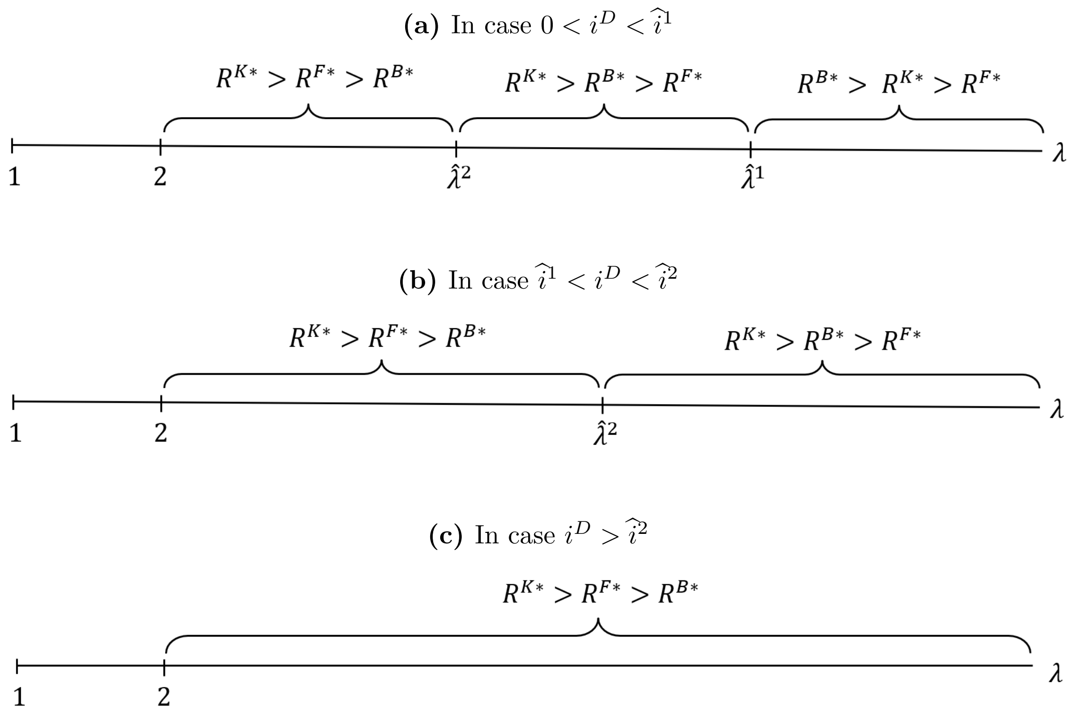
where the thresholds are defined as

$$\widehat{i}^1 = \frac{\overline{y}}{2} - \frac{w}{\tau}, \quad \widehat{i}^2 = \overline{y} - \frac{w}{\tau}, \quad \widehat{\lambda}^1 = \frac{4\overline{y}}{\overline{y} - 2\left(i^D + \frac{w}{\tau}\right)}, \quad \widehat{\lambda}^2 = \frac{2\overline{y}}{\overline{y} - \left(i^D + \frac{w}{\tau}\right)} \quad \textit{with} \ \ \widehat{i}^1 < \widehat{i}^2, \quad \widehat{\lambda}^1 > \widehat{\lambda}^2.$$

Part (i) of proposition 1 suggests that the financial sector is unambiguously dominated by the industrial sector in terms of profitability. On the other hand, the rest of the proposition suggests that the relationship between the industrial and banking sectors and that between the banking and financial sectors depend on two critical factors, i.e., the funding costs for the banks i^D and the leverage ratio of the industrial sector λ.

More specifically, part (ii) states the following. When the banks' funding costs i^D is at least as high as some threshold $\widehat{i}^1$, the industrial capital will dominate the banking capital in terms of profitability. It implies that the financial intermediation technology is not so advanced as to enable the banks to raise funds at low enough costs to beat the industrial capital. Things change when i^D is below the threshold, implying that the financial intermediation technology of the banking sector is now sufficiently strong to avoid outright dominance of the industrial capital. However, part (ii) continue to suggest that even in that case the industrial sector can still dominate the banking sector as long as its borrowing is not so high, i.e., below some threshold $\widehat{\lambda}^1$. It implies that since the industrial sector's borrowing is the source of the bank profits, limiting it below the threshold will make the banking sector disadvantageous vis-á-vis the industrial sector even when the banks have sufficiently advanced financial intermediation technology to ensure low enough funding costs. On the other hand, if the industrial sector's borrowing is sufficiently big as well as the banks' funding costs are

Figure 2: Proposition 1



sufficiently low, then the return on equity of the total banking capital will be greater than that of the total industrial capital.

When understanding the relation between the banking sector and the financial sector reported in part (iii), it needs to be kept in mind that the financial sector includes the banking sector. That said, the result of part (iii) can be understood in a similar way as in part (ii). First, there is another threshold $\widehat{i}^2 > \widehat{i}^1$ for i^D above which the funding cost is so high that it presses the banking sector profitability down below the financial sector profitability. The reason for this result of the high bank funding cost is because while i^D is a subtraction factor for the banking sector profitability, its effect on the financial sector profitability is zero sum since the banks' funding costs are the revenues for the bank creditors. Even when i^D is below the threshold $\widehat{i}^2$ the bank profitability will not rise above the financial sector profitability unless the industrial sector borrowing λ—the source of bank profits—is sufficiently large, above a threshold $\widehat{\lambda}^2 < \widehat{\lambda}^1$. But when i^D is below $\widehat{i}^2$ and, simultaneously, λ is above $\widehat{\lambda}^2$, the banking sector will dominate the financial sector.

The three parts of proposition 1, each of which being discussed above separately, can be combined together to provide overall orderings of the three types of capital. This is visualized in figure 2. From the two thresholds $\widehat{i}^1$ and $\widehat{i}^2$ emerge three regions of i^D and in each region, different regions of λ emerge. One relationship common to all of these different regions is that the industrial sector profitability is consistently greater than the financial sector profitability which is what part (i) of proposition 1 suggests. Also note that $\lambda > 2$ is the viable space due to part (i) of assumption 2, which ensures $\widehat{y}^* > 0$.

First consider the low region of i^D in figure 2a which corresponds to advanced financial intermediation technology enabling the banks to raise funds at low costs. In this case, from the two thresholds $\widehat{\lambda}^1$ and $\widehat{\lambda}^2$ emerge three different regions of λ. In the high region $\lambda > \widehat{\lambda}^1$, combined with the low i^D, the conditions are the most conducive to the banks and hence the banking profitability dominates both the industrial and financial sectors. In the mid-range region of industrial borrowing $\widehat{\lambda}^2 < \lambda < \widehat{\lambda}^1$, the banking sector dominates only the financial sector but not the industrial capital. In the low region of industrial borrowing $2 < \lambda < \widehat{\lambda}^2$, the banking sector is dominated not only by the industrial sector but also by the financial sector.

Next consider the mid-range region of i^D in figure 2b which corresponds to a mid-range level of financial intermediation technology. In this case, only the threshold $\widehat{\lambda}^2$ is relevant from which two regions of λ emerges. Even when the industrial leverage is in the high region $\lambda > \widehat{\lambda}^2$ the banking sector does not dominate the industrial sector but only the financial sector. In the low region of industrial borrowing $\lambda < \widehat{\lambda}^2$, even the financial sector dominates the banking sector.

Lastly, consider the high region of i^D in figure 2c which corresponds to weak financial intermediation technology. In this case, the financial technology is so weak that the banking sector is consistently dominated by both the industrial and financial sectors regardless of the industrial borrowing λ.

It is important to note that the results thus far are derived under the scenario that the banks provide the loan services only to the industrial sector. In the next section, we extend the model to have the banks expand their lending business to the household sector to diversify their portfolio and the sources of their income.

3 The worker-household debt

In this section, the household sector in need of borrowing is explicitly introduced. In addition to the capitalist and bankers, there are a sufficiently large number of workers. Hence, the labor supply is unconstrained and the employment is demand-driven. The total number of workers employed is K/τ where K is the total productive assets. Since the scale of productive business is λ, which each industrial capitalist finances with own wealth and bank loan, the productive asset of each industrial capitalist is λ. Therefore, the total productive asset is obtained by summing λ across all the industrial capitalists as follows.

$$K \equiv \int_{\widehat{y}}^{\infty} \lambda \mathrm{d}F(y). \tag{17}$$

In addition to wage income w, which covers expenditures for daily living staples, each worker also needs lump-sum funds for cars, housing, and education, etc. Accordingly, the worker households have to rely on bank loans which are available at the interest rate of i^H. In contrast, the capitalist households have both wealth and income which are assumed to be sufficiently large so that they do not have to incur debt; it is also assumed that agents do not borrow for financial investments. While the industrial capitalists borrow from the bank with their wealth posted as collateral, since the worker households have no wealth they have to post their future cash flows of wage income as collateral when borrowing from a bank. Which means that only those who are employed can borrow. We suppose that the unemployed are supported by the government transfers which is not explicitly formulated in the model.

Recall that in the benchmark model without household debt in section 2, wages are fixed at a level that supports the consumption required to maintain the basic standard of living, and this is due to the Classical assumption of unconstrained supply of labor. That is, denoting the consumption by c, it holds that $w = c$ at a fixed level. On the other hand, in the extended model in this section where the consumption required to maintain the basic standard of living additionally includes housing, education, etc. whose costs require incurring debt, $w > c$ should hold since the workers now have to make interest payments out of wages in addition to consumption expenditure. But due to the assumption of unconstrained supply of labor, both w and c are still fixed constant. c is fixed at a level that reflects the basic standard of living, which includes not only foods and clothing but also housing and education, whereas w is fixed at a level that reflects not only costs for foods and clothing but also interest expenses associated with debt for housing and education.

The worker households are all identical in terms of the number of family members and only one of them being a job-seeker. The worker households are debt-ridden and therefore they have no room for savings or investments. In all, the worker household's budget constraint is as follows.

$$w = c + i^H L^H \tag{18}$$

It suggests that given w and c, each worker household's interest payment is $i^H L^H = w - c$, and therefore, when the household loan interest rate is higher (lower) the household will reduce (increase) the loan.

3.1 The market for industrial loans

Now that the banks make loans to both the industrial sector and the worker household sector, the equilibrium will be different from the one established in section 2.2, where the banks make the industrial loans only. Let us suppose that for all banks the share of loans to the industrial sector is θ while the share of loans to the household sector is $1 - \theta$. Then in the industrial loan market, the total supply is

$$\theta \left(1 + \int_0^{\widehat{y}} \mathrm{d}F(y) \right) \tag{19}$$

while the demand is the same as in the benchmark model without household debt, i.e.,

$$\int_{\widehat{y}}^{\infty} (\lambda - 1) \mathrm{d}F(y). \tag{20}$$

Before we establish the equilibrium of the extended model in this section, to distinguish it from the equilibrium of the benchmark model in the previous section, we note that the superscript double star is used to indicate the equilibrium of the extended model.

By solving the integrals of equations (19) and (20), the equilibrium interest rate of industrial loan is obtained from the equality of demand and supply as follows.

$$i^{K**} = \frac{1}{\lambda - 1} \left(\lambda \overline{y} - \lambda \frac{w}{\tau} - i^D \right) - \frac{2\lambda\theta\overline{y}}{(\lambda - 1)(\lambda - 1 + \theta)} \tag{21}$$

It is easily verified that $\theta = 1$ establishes the equivalence between the benchmark and extended models in equilibrium, yielding $i^{K*} = i^{K**}$; and also that as θ declines away from 1 its upper bound towards 0 its lower bound, i^{K**} starts to increase, thereby establishing $i^{K**} > i^{K*}$ for $\theta < 1$, i.e., the industrial loan interest rate is greater in the model where the banks lend to both the industrial and household sectors than that in the model where the banks lend only to the industrial sector.

The negative correlation between i^{K**} and θ is summarized as follows.

Lemma 8 $\frac{\partial i^{K**}}{\partial \theta} < 0$.

The comparative statistic analysis of i^{K**} with respect to the other parameters is the same as in the benchmark case reported in lemma 2; lemma 8 reports the result for θ only as it is newly introduced in the extended model. All of this is intuitive since when the banks direct more loans to the industrial sector, which implies a increase in the supply of industrial loans, the industrial loan interest rate will fall.

Substituting the new equilibrium industrial loan interest rate to equation (3) yields the equilibrium threshold level of y as follows.

$$\widehat{y}^{**} = \left(\frac{\lambda - 1 - \theta}{\lambda - 1 + \theta} \right) \overline{y} - \frac{w}{\tau}. \tag{22}$$

For the same reason for adopting assumption 2, let us confine the analysis in this section to $\widehat{y}^{**} > 0$ and the total profits produced by the aggregate productive assets in the industrial sector being positive. To ensure these, we adopt following assumption on the paramter space, in place of assumption 2.

Assumption 3 *(i)* $\lambda > 1 + \theta$, *(ii)* $y = \frac{2w}{\tau}$.

Part (i) of assumption 3 ensures $\widehat{y}^{**} > 0$ which is easy to verify, and under the condition of part (i), part (ii) ensures that the total profits produced by the aggregate productive assets in the industrial sector are positive, which will become clearer in the discussion below. Again, $\theta = 1$ brings us back to the benchmark case.

The comparative statistic analysis of $\widehat{y}^{**}$ with respect to all the other parameters is the same as in the benchmark case in lemma 3; hence, lemma 3 considers only θ.

Lemma 9 $\frac{\partial \widehat{y}^{**}}{\partial \theta} < 0$

which suggests that as the banks supply more loans to the industrial sector and less to the household sector, the industrial capitalists will increase and the financial capitalists will decrease in equilibrium. This is obvious because the banks allocating a greater share of any given funds to the industrial sector will allow those financial capitalists at the margin to optimally switch to the industrial sector.

3.2 The market for household loans

The market for household loan is newly introduced in the extended model, and we can examine the market equilibrium in the same way as that of the industrial loan market. Recall that for all the banks, the share of the housing loans is $1-\theta$, and hence the supply of bank loans to the worker household sector is

$$(1-\theta)\left[1+\int_0^{\widehat{y}} \mathrm{d}F(y)\right] \tag{23}$$

and the total demand is $L^H K/\tau$, which, using equations (18) and (24), is expressed as follows.

$$\frac{w-c}{i^H}\left[\frac{\int_{\widehat{y}}^{\infty} \lambda \mathrm{d}F(y)}{\tau}\right]. \tag{24}$$

A couple of comments are necessary regarding the market for household loan. It is one of the central properties of the extended model that the equilibrium of the household loan market is ultimately driven by how the capitalist class is divided between the industrial and financial capitalists. The industrial capitalists, on the one hand, by operating the productive business activity, hire workers, and the latter are the ones who borrow from the banks and hence form the demand for household loans; recall that the unemployed are not eligible to borrow. On the other hand, the financial capitalists are the ones who provide funds to the banks by purchasing the latter's debt.

More specifically, consider, for instance, an increase in $\widehat{y}$ which implies a decrease in the number of industrial capitalists and an increase in the number of financial capitalists. The consequences are, as verified in equation (23), an increase in the supply of household loan for any given θ and, as verified in equation (24), a decrease in the total productive capital, which in turn reduces the employment of workers and eventually lowers the demand for household loans.

In this framework, when establishing the equilibrium of the household loan market the equilibrium of the industrial loan market, and hence the equilibrium threshold $\widehat{y}^{**}$, needs to be considered. Accordingly, the equilibrium interest rate of household loan is obtained by solving the integrals of equations (23) and (24) with the equilibrium threshold $\widehat{y}^{**}$ in equation (22) and equating the two.

$$i^{H**} = \frac{\lambda\theta(w-c)}{\tau(\lambda-1)(1-\theta)}. \tag{25}$$

Since i^H is a newly introduced variable in the extended model, the comparative statistic analysis of it is conducted with respect to all the relevant paramters as follows.

Lemma 10 *(i)* $\frac{\partial i^{H**}}{\partial w} > 0$, *(ii)* $\frac{\partial i^{H**}}{\partial c} < 0$, *(iii)* $\frac{\partial i^{H**}}{\partial k} < 0$, *(iv)* $\frac{\partial i^{H**}}{\partial \lambda} < 0$, *(v)* $\frac{\partial i^{H**}}{\partial \theta} > 0$.

All the results in lemma 10 are as expected and intuitive. In part (i), a higher wage implies more collateral which enables the worker households to borrow more thereby increasing the demand and hence the household loan interest rate. The opposite sign value for c in part (ii) can be understood using the same reasoning. In part (iii), a higher capital-labor ratio τ implies a less employment of workers, which then leads to a reduction in the demand for loans and hence in the household loan interest rate. In part (v), when θ rises meaning that the banks lend more to the industrial sector and less to the household sector, the supply of household loan falls and therefore the household loan interest rate rises.

Part (iv) needs some explanation as the industrial sector's borrowing λ affects i^{H**} via various intermediate channels. The demand, on the one hand, is affected as follows. First, an increase in λ raises the equilibrium threshold $\widehat{y}^{**}$; second, consequently, the number of industrial capitalists is now smaller and although each individual industrial capitalist now operates the productive business of a bigger scale—due to the increase in λ—the net effect is a decrease in the total productive assets in equilibrium, K^{**}, which can be verified by solving the integrals in equation (17) in equilibrium; third, given the capital-labor ratio τ, the decrease in K^{**} implies a reduction in the employment of workers; lastly, since only the employed can borrow with their future flows of wage income posted as collateral, the reduction of employment leads to a decrease in the demand for borrowing.

On the other hand, since the increase in $\widehat{y}^{**}$ caused by the increase in λ means an increase in the number of financial capitalists who purchase the bank debt and hence supply the funds to the banking sector, it will lead to an increase in the supply of total bank loans, and, for any given θ, it will also lead to an increase in the supply of the household loans. In all, since the rise in λ reduces the demand and raises the supply, it is evident that the equilibrium interest rate of household loan declines, which is what part (iv) suggests.

3.3 The profitability of capital

As a comparison to the capital profitability of the benchmark model without household debt in section 2.3, we present an analysis of it in the extended model with the household debt.

Let us start with the profitability of the total productive capital. The definition of the latter is the same as in the benchmark model which is in equation (9). Solving the latter along with the new equilibrium interest rate of industrial loans in equation (21) yields the equilibrium profit rate of the total productive capital as follows.

$$R^{**} = \left(\frac{\lambda - 1}{\lambda - 1 + \theta}\right) \overline{y} - \frac{w}{\tau} \tag{26}$$

The definition of the return on equity of the total industrial capital is also the same as in the benchmark model in equation (11). Hence, in a similar way, its equilibrium level is obtained as follows.

$$R^{K**} = \left(\frac{\lambda\theta}{\lambda - 1 + \theta}\right) \overline{y} + i^D. \tag{27}$$

The definitions of the return on equity of the total banking capital and that of the total financial capital are different from the benchmark case since the banking and financial sectors now have two separate flows of interest revenues from the industrial and household sectors. In contrast to equation (13), the return on equity of the total banking capital is now

$$R^B \equiv \theta(1 + D)i^K + (1 - \theta)(1 + D)i^H - i^D D \tag{28}$$

which in equilibrium is solved to

$$R^{B**} = \frac{2\lambda\theta}{\lambda - 1 + \theta} \left[\left(\frac{\lambda - 1 - \theta}{\lambda - 1 + \theta}\right) \overline{y} - \frac{c}{\tau}\right] - i^D. \tag{29}$$

Similarly, in contrast to equation (15), the return on equity of the total financial capital is now

$$R^F \equiv \frac{\theta(1 + D)i^K + (1 - \theta)(1 + D)i^H}{1 + D}. \tag{30}$$

Note that $R^F = i^K$ does not hold any longer as in the benchmark model without the household borrowing; see equation (15).[2] As is evident in equation (30), this is because of the new source of income for the financial sector, i.e., the interest payments from the household sector, which comes at a different rate, i^H. The equilibrium R^F is obtained as

$$R^{F**} = \frac{\lambda\theta}{\lambda - 1} \left[\left(\frac{\lambda - 1 - \theta}{\lambda - 1 + \theta}\right) \overline{y} - \frac{c}{\tau} - \frac{i^D}{\lambda}\right] \tag{31}$$

which is also different from i^{K**}.

Among the parameters that affect these sectoral ROEs, the impact of c is imparted through affecting the worker household's interest payments, $i^H L^H$, due to the household budget constraint in equation (18). In this context, to more directly observe how $i^H L^H$

[2] It can be verified again that $\theta = 1$ recovers the benchmark case of $R^F = i^K$.

affects the three ROEs, let us replace c by $w - i^H L^H$ and rank them with the conditions of the ranking organized around $i^H L^H$.[3] The result is as follows.

Proposition 2 *In the model economy under the parameter spaces described in assumptions 1 and 3, the following relationships hold among the returns of equity of the industrial capital, banking capital, and financial capital*

(i) $R^{K**} > R^{F**}$

(ii) $R^{K**} \gtreqless R^{B**} \iff \frac{i^{H**}L^{H**}}{\tau} \lesseqgtr \Omega^1$

(iii) $R^{B**} \gtreqless R^{F**} \iff \frac{i^{H**}L^{H**}}{\tau} \gtreqless \Omega^2$

where

$$\Omega^1 = \frac{w}{\tau} + \left(\frac{\lambda - 1 + \theta}{\lambda\theta}\right) i^D - \left(\frac{\lambda - 1 - 3\theta}{2(\lambda - 1 + \theta)}\right) \overline{y}$$
$$\Omega^2 = \frac{w}{\tau} + \left(\frac{\lambda - 1 + \theta}{\lambda\theta}\right) i^D - \overline{y}$$

with $\Omega^1 > \Omega^2$.

Proposition 2 suggests that the comparison among the three sectoral returns on equity depends on how high or low the total interest payments on the household loans per productive capital is. One exception is in part (i) which suggests that the industrial sector profitability unambiguously dominates the financial sector profitability.

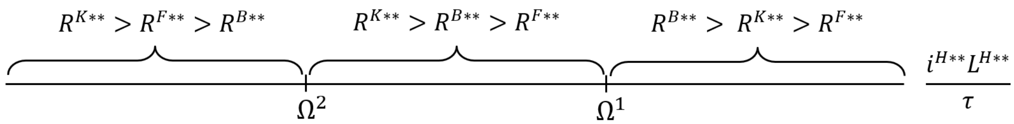
Part (ii) suggests that the banking sector profitability dominates the industrial sector profitability as long as their interest earnings from the household sector exceeds some threshold Ω^1; otherwise, the industrial sector will dominate the banking sector. The sign value of Ω^1 is indeterminate and it depends on the parameters. Suppose $\Omega^1 < 0$ due to

$$i^D < \frac{\lambda\theta}{2(\lambda - 1 + \theta)} \left[\left(\frac{\lambda - 1 - 3\theta}{\lambda - 1 + \theta}\right) \overline{y} - \frac{2w}{\tau}\right] \tag{32}$$

which is obtain by rearranging $\Omega^1 < 0$ in terms of i^D. In this case, according to part (ii) of proposition 2 $R^{B**} > R^{K**}$ will hold regardless of $i^{H**}L^{H**}/\tau$ since the latter cannot be negative. In other words, if the financial intermediation technology reflected in i^D is

[3] In order to make the theoretical results here more applicable empirically in the next section, the conditions of the ranking is organized around $i^H L^H/\tau$ instead of $i^H L^H$; the former is the total interest payments by the entire worker household normalized by the total productive capital. The main insights do not change even when the result is presented in terms of $i^H L^H$.

Figure 3: The profitabilities of industrial capital (R^{K**}) vs. banking capital (R^{B**}) vs. financial capital (R^{F**}) when the banks lend to both the industrial and worker household sectors: Visualization of proposition 2



sufficiently advanced to satisfy inequality (32), then the banking sector profitability will unambiguously dominate the industrial sector profitability regardless of how high or low their interest revenues from the household sector are.

Part (iii) on the bank sector versus the financial sector can be understood in the same way. The reason why the interest payments on the total household loans exceeding a certain threshold benefits the banking sector, not the financial sector, is because the bank creditors earn at a fixed rate i^D whereas the bankers are residual claimants.

Combining the three parts of proposition 2 together generates the consistent orders of magnitude among the three sectoral profitabilities that depend on the total interest payments by the household sector. This is visualized in figure 3. From the two thresholds Ω^1 and Ω^2 three different regions of $i^{H**}L^{H**}/\tau$ emerge. In the high region of the total interest payments by the household sector $i^{H**}L^{H**}/\tau > \Omega^1$, the banking sector profitability is the strongest and the industrial sector profitability comes next since the financial sector profitability is always weaker than the industrial sector. In the mid-range region $\Omega^2 < i^{H**}L^{H**}/\tau < \Omega^1$, the industrial sector profitability is the strongest but the banking sector still dominates the financial sector. In the low region $i^{H**}L^{H**}/\tau < \Omega^2$, while the dominance of the industrial sector remains the same, the banking sector is now dominated by the financial sector; this is intuitive since, again, the total interest revenues from the household sector is too low.

Suppose $\Omega^1 < 0$. Then since $\Omega^1 > \Omega^2$, it will also hold that $\Omega^2 < 0$. In that case, as is evident from the figure the high region will be the only viable region with $i^{H**}L^{H**}/\tau > 0$. This confirms the earlier discussion that as long as the bank funding cost i^D is sufficiently low to satisfy inequality (32) the banking sector profitability will clearly dominate the rival sectors regardless of their interest earnings from the household loans.

An important implication in relation to the results of the benchmark model without the household borrowing is the following. Recall from proposition 1 when the banks' lending businesses were only vis-á-vis the industrial sector, there were two conditions under which the profitability of the banking capital is always dominated by that of the industrial capital; either sufficiently weak intermediation technology of the banking sector or sufficiently low leverage ratio of the industrial sector. In this context, proposition 2 suggests that even under these conditions, if the banks' loans are extended to the worker households for new sources of bank revenues, the banks can possibly dominate their industrial rivals when the bank revenues from the new source is sufficiently large either due to a high interest rate on the household loans or a large volume of the household loans.

To more directly see how the worker household's interest payments positively affect the bank profitability exceeding the industrial profitability, let us express the ROE differential, $R^{B**} - R^{K**}$, between the two sectors using equations (29) and (27) with c in equation (29) replaced by $w - i^H L^H$.

$$R^{B**} - R^{K**} = \lambda\theta \left[\frac{2(i^{H**} L^{H**} - w)}{\tau(\lambda - 1 + \theta)} + \frac{(\lambda - 1 - 3\theta)\overline{y}}{(\lambda - 1 + \theta)^2} \right] - 2i^D \tag{33}$$

It is evident that $R^{B**} - R^{K**}$ is positively correlated with $i^{H**} L^{H**}$.

The results presented thus far suggest that the worker household's interest payments can be a potential source of the bank profitability exceeding the industrial profitability. This idea is empirically tested in the next section.

4 An empirical analysis

Equation (33) suggests that the ROE differential between the banking capital and the industrial capital is a function of the worker households' interest payments normalized by industrial capital, $i^H L^H/\tau$, the share of bank loans to the industrial sector, θ, the industrial sector's leverage ratio, λ, the interest rate on the bank deposit, i^D, and the maximal capital productivity, $\overline{y}$. To simplify the notations for the empirical analysis below, we denote $R^{BK} = R^B - R^K$ and $I^H = i^H L^H/\tau$. Then the function for R^{BK} can be expressed as[4]

$$R^{BK} = f(I^H, \theta, \lambda, i^D, \overline{y}) \tag{34}$$

In this section, the above relation is empirically tested relying on a cointegration analysis.

[4] Due to the household budget constraint in equation (18), a change in $i^H L^H$ already reflects a change in either w or c. Therefore, w is not included in function f in equation (34).

4.1 Methodology

For the cointegration analysis I use the bounds testing of the autoregressive distributed lag (ARDL) approach proposed in Pesaran et al. (2001). It has several advantages over the other cointegration approaches, but its most important advantage for our purpose is that it is suitable when variables integrate in different orders between $I(0)$ and $I(1)$.

A model specification of ARDL$(p, q_1, \ldots, q_k)$ is as follows.

$$Y_t = a_0 + \Sigma_{i=1}^{p} \alpha_{yi} Y_{t-i} + \Sigma_{i=0}^{q_1} \alpha_{x_1 i} X_{1,t-i} + \cdots + + \Sigma_{i=0}^{q_k} \alpha_{x_k i} X_{k,t-i} + e_t \tag{35}$$

where e_t is a white noise process, in which case the ARDL model can be estimated consistently by ordinary least squares. With some manipulations, equation (35) can be rearrranged into an unrestricted error-correction model (ECM) of ARDL as follows.

$$\Delta Y_t = a_0 + \pi_0 Y_{t-1} + \pi_1 X_{1,t-1} + \cdots + \pi_k X_{k,t-1} + \Sigma_{i=1}^{p-1} \psi_{yi} \Delta Y_{t-i} + \Sigma_{i=0}^{q_1 -1} \psi_{x_1 i} \Delta X_{1,t-i} + \cdots + \Sigma_{i=0}^{q_k -1} \psi_{x_k i} \Delta X_{k,t-i} + e_t \tag{36}$$

where Δ is a difference operator. Or, it can be alternatively expressed as a restricted ECM as follows.

$$\Delta Y_t = a_0 + \pi_0 Z_{t-1} + \Sigma_{i=1}^{p-1} \psi_{yi} \Delta Y_{t-i} + \Sigma_{i=0}^{q_1 -1} \psi_{x_1 i} \Delta X_{1,t-i} + \cdots + \Sigma_{i=0}^{q_k -1} \psi_{x_k i} \Delta X_{k,t-i} + e_t \tag{37}$$

where $\pi_0 Z_{t-1}$ is the error correction term with

$$Z_{t-1} = (Y_{t-1} - \beta_1 X_{1,t-1} - \cdots - \beta_k X_{k,t-1}) \tag{38}$$

$\beta_1 = -\pi_1/\pi_0, \cdots, \beta_k = -\pi_1/\pi_k$ hold and these are the coefficients that reflect the long-run equilibrium relationships among the variables; Z_{t-1} measures a deviation from the long-run equilibrium, π_0 is the speed of adjustment of Y_t to correct the deviation and converge back to the equilibrium; ψ_{yi} and ψ_{xi} are short-run elasticities. Within the ARDL bound test framework, all the X's are weakly exogenous, i.e., they do not contain any useful information in making an inference on Y, or X's do not respond to any deviation from the long-run equilibrium.

As proposed in Pesaran et al. (2001), there are two steps involved in the bounds test for cointegration based on the asymptotic critical bounds value. The first is the F-test, or the Wald test, where the joint null hypothesis is $H_0^F : \pi_0 = \cdots = \pi_k = 0$ against the joint alternative of $H_1^F : \pi_0 \neq 0$ or, $\cdots$, or $\pi_k \neq 0$. A lower bound associated with the assumption that all variables are $I(0)$ and an upper bound associated with the assumption that all variables are $I(1)$ can be computed. If the F-statistic is less than the lower bound the test fails to reject the null; if the F-statistics is greater than the upper bound the null is rejected; if the F-statistics is between the two bounds, the result is inconclusive.

In case it fails to reject the null, proceed no further and conclude no cointegration. In case the null is rejected, however, an additional bounds t-test needs to be performed to verify whether the coefficient of the speed of adjustment is zero. The null is H_0^t: $\pi_0 = 0$ against the alternative H_1^t: $\pi_0 < 0$. The lower and upper bounds associated with the model specification are computed. If the t-statistics is less than the absolute value of the lower bound the test fails to reject the null, eventually indicating no cointegration. If the t-statistics is greater than the absolute value of the upper bound the null is rejected, eventually indicating the cointegrating relationship between the two variables. If the t-statistics is between the two, the result is inconclusive.

4.2 Data

The data of the return on equity of the banking sector and the industrial sector (nonfinancial corporate business) are from the Federal Financial Institutions Examination Council and the Z.1 Financial Accounts of the United States, respectively. The leverage ratio of the industrial sector (nonfinancial corporate business) is also computed from the Z.1 Financial Accounts of the United States. For the total wages, we used wages and salaries of the private sector from the National Income and Product Account. For the interest rate on bank deposit, we used the prime rate data from the Fed's H.15 Selected Interest Rates.

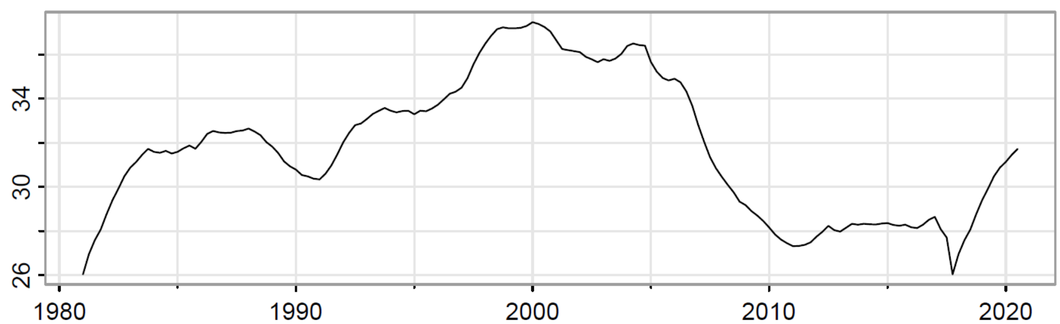
The empirical analysis uses the quarterly data of the U.S. economy for 1984Q1-2020Q3. The source data for the return on equity of the banking sector and the nonfinancial corporation sector are from the Z.1 Financial Accounts of the United States and the returns on equity of the two sectors are presented in figure 1. The differential between the two, which is the dependent variable, is illustrated in figure 4a.

Some comments on constructing the time series of I^H are necessary. First, it is the worker households' debt payments, but we proxy it by considering the entire household sector. For this, the following data are used: the household Debt Service Ratio (DSR), which is the household debt payments (HDP) as a share of disposable personal income (DPI),

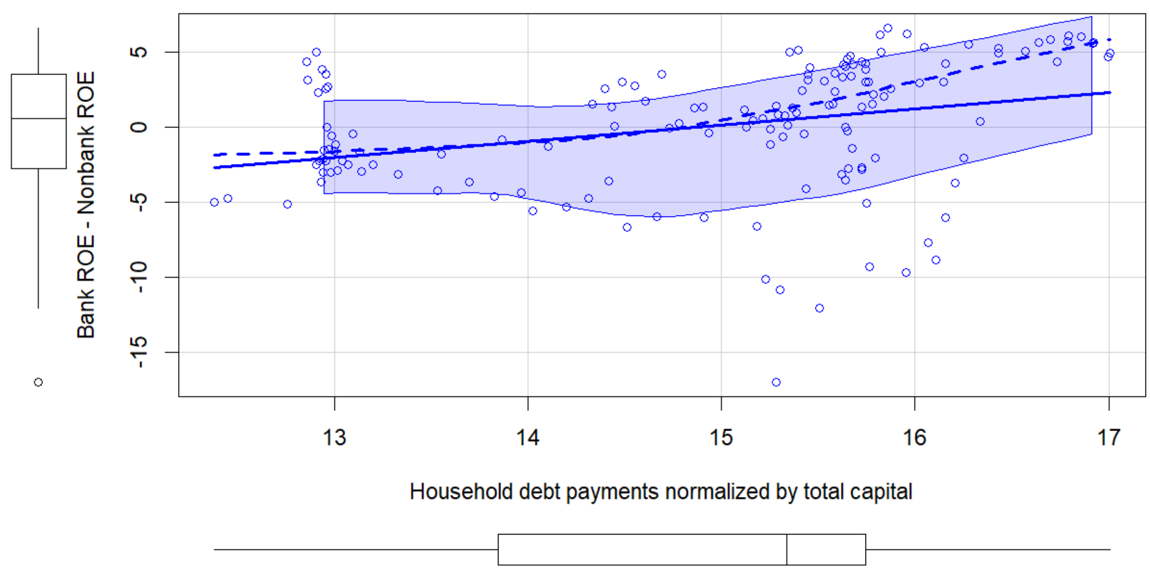
Figure 4: 1984,Q1–2020,Q3, quarterly

(a) $R^B - R^K$

(b) $i^H L^H / \tau$



(c) Scatterplot



and nonfinancial assets of nonfinancial corporate business (NFA). Hence, I^H is estimated as follows:

$$\frac{i^H L^H}{\tau} \equiv \frac{HDP}{NFA} = \frac{DSR \cdot DPI}{NFA} \tag{39}$$

The time series of HDP/PFA is presented in figure 4b.

A similar trend is observed between the two time series before and after around 2000. Prior to 2000, a long period of increasing trend is observed despite some ups and downs and a huge dip of $R^B - R^K$ in 1987, and after 2000 a stable movement with a slightly falling trend is observed through 2008, after which a precipitous fall follows due to the Global Financial Crisis; and afterwards, some increasing trends starts although when it starts is different between the two variables. The correlation between the two variables for the entire period is better illustrated in a scatterplot in figure 4c.

4.3 Results

To examine the stationarity of the variables, we conducted the augmented Dickey-Fuller (ADF) test and the Philips-Perron (PP) test. Both have the null hypothesis of unit root. For the ADF test, the lag order of each variable is determined by the information criteria of Akaike information criterion (AIC) and Schwarz information criterion (SIC). The results are reported in table 1. As for the ROE differential in level, on the one hand, the null of unit root is rejected at the 1% significance level in the ADF test while at the 5% significance level. As for the rest of the variables in level, on the other hand, the null of unit root is failed to be rejected in either test, but in the case of their first difference the null is rejected at the 1% significance level in both tests. In all, these results suggest that the ROE differential is $I(0)$ process and hence stationary while all the other variables are $I(1)$ process and hence nonstationary.

Since the variables are a mix of $I(0)$ and $I(1)$, we can conduct the ARDL bounds test. The results are reported in table 2. Starting with the F-test, the F-statistic is 4.5627, which exceeds the upper bound at the 5% significance level and hence rejects the null of no cointegration. Hence, we moved onto the t-test, and t-statistic is -4.3203, which exceeds the upper bound in absolute term at the 5% significance value. In all, these results suggest that the variables are cointegrated forming a long-run equilibrium relationship.

Next, table 3 reports the estimation of the ECM in equations (36) and (37) which measures

Table 1: Unit root test results

		ADF test		PP test	Decision
		AIC	BIC		
R^{BK}	Level	-2.6008^{***}	-2.6274^{***}	-3.1071^{**}	$I(0)$
I^H	Level	-0.7342	-0.7342	-1.6442	$I(1)$
	First difference	-2.7709^{***}	-2.7709^{***}	-4.1241^{***}	
θ	Level	-0.5773	-0.5773	-1.5198	$I(1)$
	First difference	-5.2015^{***}	-5.2015^{***}	-6.8647^{***}	
λ	Level	0.0935	0.2481	-2.2153	$I(1)$
	First difference	-4.8693^{***}	-6.9087^{***}	-11.1134^{***}	
i^D	Level	-1.867	-2.9809^{**}	-1.935	$I(1)$
	First difference	-5.7964^{***}	-5.7964^{***}	-6.665^{***}	

Note: $***$, $**$, and $*$ indicate the significance level at 1%, 5%, and 10%

Table 2: Bounds test result for cointegration

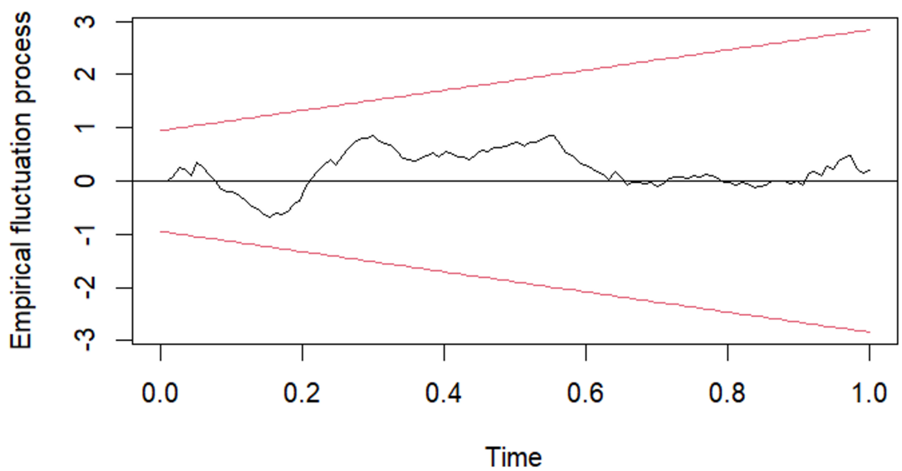
F-test	F-statistic (p-value)	Critical bounds value (5%)	
		Lower bound $I(0)$	Upper bound $I(1)$
	4.5627^{**} (0.0265)	2.9706	4.1203
t-test	t-statistic (p-value)	Critical bounds value (5%)	
		Lower bound $I(0)$	Upper bound $I(1)$
	-4.3203^{**} (0.0242)	-2.8830	-4.0105

Notes: $***$, $**$, and $*$ indicate the significance level at 1%, 5%, and 10%.

the long-run and short-run dynamics of the variables. The lag structure of the ARDL model is determined by AIC and is ARDL(5,6,0,5,3). Our main focus is on how the households' interest payments affect the ROE differential between the two capitalist sectors. First, the long run coefficient of I^H is quite large at 1.5099. It means that 1% increase in I^H leads to 1.51% increase in R^{BK}. Second, the coefficient, π_0, of the error correction term is -0.3506. The negative sign implies that R^{BK} responds to any deviation from the long-run equilibrium in a way that corrects it. Quantitatively, when there is a shock in any given period generating a deviation from the equilibrium, around 35% of it will be corrected in the following period. These are all statistically significant at the 1% level.

Lastly, table 4 reports diagnostic test results. First, the Breusch-Godfrey Test has the null hypothesis of no serial autocorrelation. The result suggests that the null is failed to be rejected and hence there is no serial autocorrelation. Second, the Breusch-Pagan test has the null of no heteroscadacity. The result suggests that the null is failed to be rejected and hence the model does not suffer from heteroscadacity. Third, the Ramsey RESET test has the null of no model misspecification. The result suggests that the null is failed to be rejected and hence the model does not suffer from misspecification. Fourth, the Recursive CUSUM test is to examine whether the estimates are stable. The result is illustrated in figure 5. As long as the black line stays within the red boundaries, which is the case; hence the model is stable.

Figure 5: Recursive CUSUM test



5 Conclusion

Both the theoretical results and the empirical application suggest that household debt and the associated debt service can be an possible source of the banking sector's profitability exceeding that of the nonfinancial corporation sector. The implication is that the banking capital beats the industrial capital in terms of profitability by exploiting the households' financial conditions that require them to rely on debt for their reproduction.

Why is the bank profitability exceeding the industrial profitability concerning? The data presented in figure 1 shows that it can possibly be correlated with an increase in the systemic risk as measured by credit-to-GDP gap. This can be one possible channel that provides an additional rationale for why there should be a policy that manages the household debt and debt service ratio at some prudent level.

Table 3: The estimation result of the restricted ECM of ADRL(5,6,0,5,3)

Regressors	Coefficient	Standard error	t-statistic	p-value
Long-run				
I^H	1.5099***	0.52642355	2.868262	0.004897631
θ	0.5680***	0.17079459	3.326096	0.001177099
λ	0.0852	0.09601845	0.886855	0.376976304
i^D	-0.7180	0.46462560	-1.545374	0.124956427
Short-run				
Intercept	-24.09634	4.96767	-4.851	0.0000
ΔR^{BK}_{t-1}	0.07948	0.09596	0.828	0.409149
ΔR^{BK}_{t-2}	0.22469**	0.09271	2.423	0.016854
ΔR^{BK}_{t-3}	-0.07393	0.09530	-0.776	0.439394
ΔR^{BK}_{t-4}	0.23366**	0.09138	2.557	0.011789
ΔI^h_t	2.92664	2.20906	1.325	0.187723
ΔI^h_{t-1}	3.16602	2.39330	1.323	0.188374
ΔI^h_{t-2}	-4.41890*	2.32234	-1.903	0.059446
ΔI^h_{t-3}	3.83868	2.34022	1.640	0.103538
ΔI^h_{t-4}	1.29687	2.29035	0.566	0.572284
ΔI^h_{t-5}	-4.55059**	2.02192	-2.251	0.026215
$\Delta\lambda_t$	-0.04074	0.05821	-0.700	0.485385
$\Delta\lambda_{t-1}$	0.01365	0.05932	0.230	0.818352
$\Delta\lambda_{t-2}$	-0.01593	0.06024	-0.264	0.791885
$\Delta\lambda_{t-3}$	-0.20995***	0.06125	-3.428	0.0008
$\Delta\lambda_{t-4}$	-0.09856	0.06137	-1.606	0.110842
Δi^D_t	-0.26844	0.57161	-0.470	0.639469
Δi^D_{t-1}	0.23480	0.64918	0.362	0.718210
Δi^D_{t-2}	0.84780	0.58774	1.442	0.151755
π_0 (ECT)	-0.3506***	0.07219	-4.857	0.0000
Residual standard error	Multiple R^2	Adjusted R^2	F-statistic	p-value
2.057 on 121 DF	0.3395	0.2358	3.273 on 19 and 121 DF	0.0000

Table 4: Diagnostic Test Results

Test method	Statistic (p-value)	Decision
Breusch-Godfrey Test	8.4277(0.07711)	No serial autocorrelation
Breusch-Pagan test	22.225(0.5067)	No heteroscedasticity
Ramsey RESET test	1.2594(0.2641)	No misspecification
Recursive CUSUM test	Figure 5	Stable

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