Optimal Inflation Targeting with Anchoring

Thomas R. Michl and Robert Rowthorn

June 2023
(Updated November 2023)

Optimal Inflation Targeting With Anchoring

Thomas R. Michl
Emeritus Professor
Colgate University

Robert Rowthorn[1]
Emeritus Professor
Cambridge University

November 1, 2023

Abstract

Optimal Inflation Targeting With Anchoring
by Thomas R. Michl and Robert Rowthorn
JEL E31
Keywords: Taylor Rule, Taylor Principle, sacrifice ratio, central bank loss
function

    This paper presents an alternative foundation to the standard quadratic loss function characterizing central bank inflation policy. The alternative treats high employment as a social benefit. In recognition of the inherent asymmetry of the output gap, two self-imposed constraints provide guardrails that rule out excess unemployment and opportunistic reflation. The loss function includes a novel reverse discounting mechanism that penalizes the bank for more sustained inflation gaps that could undermine confidence in the central bank's target. In the absence of anchoring, the central bank is obliged to use economic slack to accomplish a disinflation but the presence of anchoring creates greater policy flexibility freeing it from the tyranny of the sacrifice ratio. The central bank's optimal policy differs dramatically from the standard Taylor Rule recommendation in choosing policy plans with higher employment, in its willingness to overshoot inflation targets, and in avoiding excess unemployment, all while observing the discipline needed for successful inflation targeting.

This paper characterizes the decision problem of what we will refer to as a *socially responsible central bank* faced with temporary shocks to inflation or aggregate demand. The problems of disinflation and reflation are not as symmetrical as they appear in the conventional treatment of optimal policy summarized by the famous Taylor Rule. The microfoundation for the Taylor Rule is a quadratic loss function that treats a high-pressure labor market with employment above its inflation-neutral level as equivalent to a depressed labor market with excess unemployment.[1] The loss function pursued by our socially responsible central bank would recognize that the former represents a social benefit and plan accordingly. The paper proposes one such loss function and explores its implications for macro-policy. One innovation is that the central bank weighs losses imposed by inflation gaps (deviations from target) that are incurred in the distant future more heavily than gaps incurred in the near future. This is because persistent gaps jeopardize the confidence that embeds anchoring in the inflation process.

Because it uses the full flexibility made possible by inflation anchoring the optimal policy winds up being dramatically at odds with the plans recommended by a standard Taylor Rule derived from a quadratic loss function. This efficiency gain shows up concretely in greater attention to employment gains, principled resistance to using excess unemployment to speed up disinflation, and more tolerance for temporary inflation gaps, all without giving up the discipline required of an inflation-targeting regime. In our model the central bank optimizes with some constraints chosen through its own context-dependent assessments of the costs and benefits of output and inflation gaps, including the risk that persistent breaches of these constraints could jeopardize the degree to which inflation is anchored to the policy target. Like the medical arts, central banking combines judgment and rigor.

1   Background

In this paper, the central bank operates in a standard three-equation environment. The three-equation model which informs both economic pedagogy and policy formation can actually be reduced to a two-equation model: the Phillips curve and the central bank's loss function form a self-contained system. These two equations characterize the sequence of optimal levels of output and inflation rates that we will call the *policy plan*. The third equation, the IS-curve, can be partitioned off to provide guidance on how to achieve a given policy plan through interest rate policy. The paper focuses on the policy plan.

We make the following assumptions: First, there is a well-defined and unique inflation-neutral level of output and employment that does not respond to temporary shocks; hysteresis or path dependence is absent.[2] Second, the central

---

[1] For an accessible explanation of the standard approach, see Carlin and Soskice (2015, Chs. 3, 13). More extensive discussion can be found in Woodford (2003).

[2] For the implications of the Taylor Rule in a three-equation model in the presence of path dependence, see Michl (2018). For an alternative policy rule that compensates for the presence

bank has considerable power in regulating aggregate demand with a lag so that levels of output can be chosen with confidence. Problems associated with the effective lower bound or interest rate pessimism ("pushing on a string") are put to one side. We leave open the open the option of using fiscal policy. Third, the labor force and capital stock are predetermined and remain constant through time. We focus on the intensity of their utilization and ignore log-run issues of growth and distribution.

Fourth, the inflation process is anchored to the central bank's official inflation target. This assumption is fairly well supported by empirical research on the Phillips curve (Blanchard, 2016; Ball and Mazumder, 2011) and turns out to have enormous importance for central banking. Indeed, anchoring represents a social resource that gives policy makers considerable flexibility in pursuing the twin goals of price stability and maximum employment proscribed in the U.S. by the Humphrey-Hawkins Full Employment Act.

These assumptions are widely accepted in the professional debates, policy circles, textbook presentations, and public forums that one hopes will be influenced by any results we obtain. It is noteworthy that economists like Greg Mankiw (2006) who have served in policy-making capacities report that actual deliberations do not involve highly sophisticated economic modelling or high theory: policy making is more like engineering than pure science.

The standard approach to formulating a monetary policy regime (such as the Taylor Rule) models the central bank using a quadratic loss function. The bank minimizes the weighted sum of the squared inflation and output gaps, defined relative to an inflation target, π^T, and an inflation-neutral equilibrium level of output, y_{neut}.[3] The assumption is that the central bank's interest rate policy has considerable control over the level of demand, so that the path of interest rates can be backed out (e.g. as a Taylor Rule) of the optimal policy plan for output and inflation. An expectations-augmented Phillips curve represents the constraint in this problem which in the standard approach (Carlin and Soskice, 2015, Ch. 13) is solved statically period-by-period.[4] This approach leads to the recommendation that an inflation shock needs to be absorbed by aggressive demand contraction. At best, the central bank might simply choose to reduce demand to its neutral level (if no weight is assigned to the inflation gap). But generally, this approach calls for some slack (aka unemployment) to stabilize inflation.

[Figure 1 goes here. Figures appear at end of paper]

---

of path dependence in a three-equation setting, see Michl and Oliver (2019). Both these efforts accept the standard quadratic loss function which is in question in the current paper.

[3]Formally, the loss function with equal weights on both gaps would be $(\pi_t - \pi^T)^2 + (y_t - y_{neut})^2$. Assigning different weights to these gaps leads to alternative policy rules. The original Taylor (1993) Rule was not explicitly based on optimization which only became *de rigeur* in subsequent writing.

[4]There is a dynamic optimization version that has been championed by Janet Yellen (2012), for example, but it accepts the quadratic loss function. For details of how Fed economists solve the optimal control problem, see Brayton et al. (2014). Significantly, these efforts do not use an expectations-augmented Phillips curve with anchoring and instead adopt some form of perfect foresight.

Figure 1 identifies four quadrants centered on the long-run equilibrium formed by the inflation target, π^T, and the inflation-neutral level of ouput, y_{neut}. The standard approach advocates a response to any shock that mainly restricts the policy plan to quadrants II and IV. An inflationary shock requires the central bank to choose output levels in the disinflation zone ($y_t < y_{neut}$) in quadrant II for example.

As a description of how central banks work,[5] the quadratic loss function makes some sense since their governing bodies are sensitive to political pressure when they miss their inflation target (e.g., from finance capital when they overshoot and from workers' representatives when they undershoot), and they are probably averse to running a high-pressure labor market since they believe it will eventually raise inflation and require future retrenchment. But a central bank accountable to a well-organized working class and other community organizations, say in Robert Heilbroner's "slightly imaginary Sweden," would surely reject this loss function. It treats high employment as equally costly as unemployment, ignoring that it puts workers in a good bargaining position where they can enjoy the benefits of a high-pressure labor market. It should be treated as a gross benefit and entered into the loss function with a negative sign. In other words, there is no reason to prevent the policy plan from entering quadrant I (defined to include $y = y_{neut}$) in Figure 1.

2 Model

The Phillips curve takes the following form:

$$\pi_t = \pi_t^R + \alpha(y_t - y_{neut})$$

where π is inflation, y is output, y_{neut} is the potential or inflation-neutral level of output, and α is the sensitivity of inflation to slack. Subscripts represent time in the policy plan (i.e., in an *ex ante* sense), with initial conditions specified at $t = 0$. We implicitly assume that employment depends mechanically on output through some sort of Okun's Law with constant labor productivity, and will freely use employment and output as virtual synonyms.

Inflation depends on a reference rate, π_t^R, that is normally described as the expected rate of inflation. It is preferable to call it a reference rate because we will treat it as a kind of state variable that is embedded in the practices and institutional structures (such as cost-of-living clauses) governing wage and price setting.[6] It makes some sense following Carlin and Soskice (2018), for example, to think of bargaining over wages using the last realization of inflation

---

[5]A few writers (Woodford, 2003) think of the traditional loss function as some kind of social welfare function, which does not seem very convincing given the fact that it treats high employment as a social cost. To be clear, in this paper the loss function we use reflects the judgments of the central bank and is not a social welfare function.

[6]One of the defining characteristics of the structuralist macroeconomic (Taylor, 2004) approach this paper adopts is its rejection of methodological individualism and recognition that few really important phenomena can be reduced to individual rational choice.

(modified by some anchoring) as a focal point for negotiations, even though the participants themselves may have very different private expectations, because it economizes on transactions costs.

We will model π_t^R as an adaptive process anchored to the central bank's target rate of inflation, π^T, as in

$$\pi_{t+1}^R = \chi \pi^T + (1 - \chi)\pi_t$$

where χ is the degree of anchoring and $0 \leq \chi \leq 1$. Its complement to one, $(1 - \chi)$, is sometimes called the degree of inertia. The assumption that the coefficients on the right-hand side sum to unity ensures homogeneity.[7]

We take the degree of anchoring to be a given. There is a rich literature on the evolution of institutions such as indexation which has long been seen as a source of inflation inertia that reduces central bank flexibility, particularly in Latin America (Oreiro and Santos, 2023). Our emphasis on the value of preserving anchoring is consistent with that concern.

Combining the above equations yields the following dynamic equation

$$\pi_{t+1} - \pi^T = (1 - \chi)(\pi_t - \pi^T) + \alpha(y_{t+1} - y_{neut}). \tag{1}$$

The above equation throws a light on the intrinsic stability of the inflationary process. Suppose that $\alpha(y_t - y_{neut})$ is constant and equal to c for $t \geq 1$. If $0 < \chi < 1$ the above equation has the solution

$$\pi_t - \pi^T = (1 - \chi)^t \left(\pi_0 - \pi^T - \frac{c}{\chi} \right) + \frac{c}{\chi}$$

Thus, inflation converges to $\pi^T + \frac{c}{\chi}$. Anchoring makes the inflationary process stable. Written out in terms of next period's inflation and output (the $t+1$ subscript) these equations reflect the viewpoint of a forward-looking bank which sets the interest rate today that determines (with a lag) next period's output level.

The standard loss function needs to be modified in two ways. First, as already noted the employment gap needs to enter it with a negative sign. This removes one rationale for using the quadratic form (treating both gaps symmetrically), and raises the question of how to specify the gain associated with high employment. There doesn't seem to be any harm in assuming that it is linear.[8]

On the other hand, it makes sense to keep the quadratic form for the inflation gap since large spikes in inflation are a source of distress, and because low inflation reduces the ability of the central bank to control the real interest rate. A small amount of inflation is a good thing, but not too much. The central bank target is the ideal, and deviations from it in either direction are undesirable.

---

[7]This formalization of adaptive expectations with anchoring was proposed by Carlin and Soskice (2015, Ch. 4). For empirical estimates of a Phillips curve using this treatment, see Ball and Mazumder (2011).

[8]Barro and Gordon (1983b) work with an objective (rather than a loss) function and make it linearly increasing in output which in their case leads to time inconsistent inflation.

Second, the central bank needs an incentive to avoid the temptation to persistently ignore its own target rate of inflation, thereby eroding the confidence that underwrites anchoring the inflationary process. Anchoring gives the central bank a lot of freedom to maneuver and is a public resource worth preserving.[9] Indeed, it is a little like a good credit rating: worth the effort to achieve, but only if you take advantage of it when you really need it. To this end, we can deploy a form of reverse discounting that penalizes the central bank for lingering in a high inflation zone for too long. The central bank will have to choose an entire policy path and deploy dynamic optimization methods.

A loss function that meets these requirements[10] is

$$F = \beta A(t)(\pi_t - \pi^T)^2 - (y_t - y_{neut})$$

where β is a scale or weighting factor (relative to the output gap), and $A(t)$ is a factor that penalizes high or low inflation harshly in the more distant future. Since it operates like a reverse discount factor, we will call this the *accrual factor*.[11]

An inflation gap that persists in the policy plan raises the likelihood that expectations will become de-anchored through behavioral and institutional changes, imposing greater subjective losses on the central bank. Since these losses accrue with the progression of time in the plan starting at $t = 1$, we assign the following properties to the function that governs the accrual factor:

$$A = A(t) \qquad A' > 0 \qquad A'' > 0.$$

A functional form[12] that meets these requirements and is familiar to economists is exponential growth or its discrete-time counterpart,

$$A = (1+b)^t \qquad b > 0 \qquad t = 1, 2, \ldots.$$

In implementing this loss function in a three-equation setting, we will find that the intrinsic weight assigned to the inflation gap (β) regulates the initial response of output in the policy plan while the accrual factor $(1 + b)$ regulates the speed and shapes the character of the response over the planning interval.

---

[9]One example is that in the presence of path dependence, anchoring allows the central bank some space to pursue an output target so that it can repair the damages inflicted by temporary negative shocks (Michl and Oliver, 2019).

[10]These requirements are also met by any loss function of the following form:

$$F_t = \beta A(t)(\pi_t - \pi^T)^2 - f(y_t)$$

where $f'(y_t) > 0$. We have chosen the linear form because it is simple and permits an explicit solution.

[11]Most intertemporal models of optimal monetary policy, such as the Fed's optimal control approach (Brayton et al., 2014) use a true discount factor that shrinks down future inflation and output gaps (present discounting). It is not entirely clear what justifies that treatment since it effectively rewards the bank for postponing gap closure.

[12]There is no inherent reason why the accrual factor should take a functional form so similar to an interest or profit rate. Foley et al. (2019) uses a functional form that restricts the sum of discount factors to be one.

Unlike the accrual factor, the intrinsic weight on the inflation gap does not have an unambiguous interpretation. It cannot be a measure of the cost of inflation since it applies equally to inflation rates low and high. And even if positive gaps are capturing these costs to some extent, it has to be said that the usual suspects, shoeleather costs and tax distortions, look minimal. For negative gaps, the threat afforded by the effective lower bound at least seems substantial.

It also makes some sense to impose a conditional output floor based on the Hippocratic principle "do no harm." A socially responsible central bank would not want to deliberately create any unnecessary unemployment since with anchored expectations the inflation rate will decline at the equilibrium level of output (and even above it). This self-stabilizing property of expectations is, after all, one of the main social benefits of anchoring. The constraint

$$y_t \geq y_{low}$$

formalizes the Hippocratic principle. In the most rigid interpretation, the floor might be set at the inflation-neutral level of output. This floor is conditional because there will be circumstances when it is untenable or undesirable. In effect, this constraint arises from recognition of the inherently asymmetric nature of the social costs and benefits of the output (employment) gap.[13]

Finally, it is necessary to consider restrictions that prevent opportunistic reflations of the sort that motivated the old Barro-Gordon (1983a) model of time-inconsistent inflation. We can see immediately that the central bank will be tempted to create a temporary boom even without any shocks in the initial period, $t = 0$. The loss function is minimized at a level of output greater than the inflation-neutral level.[14] The Odyssean solution is to simply prohibit any such opportunistic behavior by imposing a restriction like

$$y_t \leq y_{high}.$$

We will call this the Odyssean constraint.[15] Again, the ceiling might literally be set at the current level of output in the appropriate context. The presence of anchoring in effect implies that the central bank already has some credibility in this regard. As Alan Blinder (1998) observed after his stint on the Federal Open Market Committee, central bankers in practice avoid the temptation to inflate by obeying a self-imposed rule of thumb: "just don't do it."

Even more than the Hippocratic constraint, this restriction is clearly context-dependent since reflation will likely require an injection of demand. Indeed, the inflation bias built into the loss function might be regarded as a positive feature

---

[13]It is more or less equivalent to treating the relative weight on the inflation gap as zero for employment and output levels below the inflation-neutral values. There may be some advantage to considering functional forms that exhibit this asymmetry in a more organic fashion.

[14]To be precise, $y_1 = y_{neut} + 1/(2\beta\alpha^2)$ minimizes the loss function in period 1.

[15]Odysseus (Ulysses in Latin) was the protagonist in the Odyssey who, on the advice of the Greek god Circe, had himself lashed to the mast of his ship to avoid being lured to his death by the songs of the Sirens.

rather than a bug since it complements the required response to a negative demand shock and creates positive social benefits as well.[16]

It is worth stepping back to remark on a basic asymmetry in our model. The inflation process is grounded in institutional structures that give it an inertial quality because of the adaptive properties of the reference rate of inflation as modified by anchoring. The central bank, on the other hand, is forward looking and operates as if it had perfect foresight, taking the structural features of the inflation process for granted. This asymmetry shares some characteristics with the celebrated Dornbusch (1976) model of exchange rate overshooting which depends on the interaction of inertial product markets with forward-looking central banks and asset markets.

3 Solving the program

The bank's problem is to choose $\{y_t\}_{t=1}^{t=\infty}$ so as to minimise

$$\sum_{t=1}^{t=\infty} [\beta(1+b)^t(\pi_t - \pi^T)^2 - (y_t - y_{neut})]$$

subject to equation (1) and the following conditions

$$y_t \in [y_{low}, y_{high}]$$

$$\lim_{t \to \infty} \pi_t = \pi^T$$

with π_0, β, b, π^T, y_{low}, y_{high}, y_{neut}, χ, and α given.

It is assumed that

$$y_{low} \leq y_{neut} \leq y_{high} \tag{2}$$

$$y_{low} \leq y_{neut} + \frac{\chi[1 - (1-\chi)(1+b)]}{2\alpha^2\beta(1+b)} \leq y_{high}. \tag{3}$$

The Lagrangian for this problem is :

$$L = \sum_{t=1}^{t=\infty} -[\beta(1+b)^t(\pi_t - \pi^T)^2 - y_t]$$
$$+ \lambda_{t+1}((\pi_{t+1} - \pi^T) - (1-\chi)(\pi_t - \pi^T) - \alpha(y_{t+1} - y_{neut}))$$
$$+ u_t(y_{high} - y_t) + v_t(y_t - y_{low}).$$

The first order conditions for an optimum are as follows.

---

[16]More broadly, one of the problems identified (Michl, 2018) with the standard loss function is that in the presence of hysteresis its disinflationary bias will lead unfavorable demand or inflation shocks to have permanent negative output effects.

$$\frac{\partial L}{\partial y_t} = 1 - u_t + v_t - \lambda_t \alpha = 0$$

$$u_t \geq 0, (y_{high} - y_t) \geq 0, u_t(y_{high} - y_t) = 0$$
$$v_t \geq 0, (y_t - y_{low}) \geq 0, v_t(y_t - y_{low}) = 0$$

$$\frac{\partial L}{\partial \pi_t} = -2\beta(1+b)^t(\pi_t - \pi^T) + \lambda_t - (1-\chi)\lambda_{t+1} = 0 \qquad (4)$$

$$\lim_{t\to\infty} \pi_t = \pi^T.$$

Since $\lim_{t\to\infty}\pi_t = \pi^T$ there is no condition on $\lim_{t\to\infty}\lambda_t$. Since the minimand is convex in π_t and y_t, and the constraints are linear, any solution that satisfies the above first order conditions is optimal.

The shadow prices u_t and v_t are the subjective penalties the bank pays for abiding by the Odyssean and Hippocratic constraints. Thus, if $u_t > 0$ the bank would prefer a value of y_t greater than y_{high} but this is ruled out by the Odyssean constraint. The shadow price u_t is the price the bank would be willng to pay for relaxing this constraint by one unit. An analogous intepretation applies to v_t.

3.1 Singular path

Suppose that the limits on y_t are so wide that they impose no effective constraint on this variable. The resulting solution must be interior and hence for $t \geq 1$:

$$\lambda_t = \frac{1}{\alpha}.$$

From (4) it follows that $\pi_t = \pi_t^*$ where

$$(\pi_t^* - \pi^T) = \frac{\chi}{2\alpha\beta(1+b)^t}. \qquad (5)$$

Suppose also that

$$\pi_0 - \pi^T = \frac{\chi}{2\alpha\beta}.$$

Equation (1) can be written as follows

$$\alpha(y_{t+1} - y_{neut}) = (\pi_{t+1} - \pi^T) - (1-\chi)(\pi_t - \pi^T). \qquad (6)$$

For $t > 1$ equations (5) and (6) imply that $y_t = y_t^*$ where

$$y_t^* - y_{neut} = \frac{\chi[1 - (1-\chi)(1+b)]}{2\alpha^2\beta(1+b)^t}. \qquad (7)$$

The quantities $\pi_t^* - \pi^T$ and $y_t^* - y_{neut}$ decline geometrically towards zero. We shall refer to the trajectory of π_t^* as the *singular path*. Note that inflation on

8

this path always approaches the central bank's target from above. Output, on the other hand, will lie above or below the inflation-neutral level depending on the sign of $1 - (1 - \chi)(1 + b)$. This expression is positive if $b < \frac{\chi}{1-\chi}$. Conditions (2) and (3) ensure that all points on the singular path lie within the permitted range.

3.2 First steps

If the constraints on output permit, the following value of y_1 will be optimal and in one step will take the inflation rate π_t onto the singular path:

$$ y_1^{**} - y_{neut} = \frac{\chi}{2\alpha^2\beta(1+b)} - \frac{(1-\chi)}{\alpha}\left(\pi_0 - \pi^T\right). $$

It may be that y_1^{**} lies outside of the permitted range $[y_{low}, y_{high}]$. The solution in this case is to move as rapidly to the singular path as the constraints allow. Suppose that π_0 is so large and positive that $y_1^{**} < y_{low}$. It is then optimal to set $y_t = y_{low}$ until the singular path comes into range. Conversely, if π_0 is so large and negative that $y_1^* > y_{high}$, it is optimal to set $y_t = y_{high}$ until the singular path comes into range. Thus, subject to conditions (2) and (3), the general solution to the optimisation problem is to reach the singular path as rapidly as the constraints allow and then to remain permanently on this path.

The final term on the right-hand side of the above equation indicates the instantaneous sacrifice needed to close the entire inflation gap all at once. This enables us to interpret the actual choice of output as a response to the inflation gap modified by the shadow prices enforcing discipline on the central bank. The shadow prices and the accrual factor are effectively distributing the cumulative output gap over multiple periods. We can also characterize the optimal policy plan qualitatively using this interpretation. The farther out from the initial shock at $t = 0$ we plan, the more discipline is imposed by the accrual factor, and the closer the optimal plan will be to closing the inflation gap fully by choosing a suitable output gap.

The rest of the paper considers the central bank problem in the context of each of the circumstances represented by the four quadrants of Figure 1, paying close attention to the political economy of the constraints and the parameters of the loss function. There is no presumption that these parameters would be or should be the same in every scenario. Some of the numerical examples have been generated with an eye more toward showcasing the properties of the model than out of a sense of realism and it is hoped they will be evaluated in that spirit. Although the initial output y_0 does not affect the optimum policy, we include some mention of this variable to provide a context for our analysis.

4 Disinflation

We begin in quadrants I or II with inflation elevated either because of a pure temporary inflation shock (an exogenous supply shock), a pure demand shock,

or some combination. What matters most to the central bank is the initial inflation rate since that establishes the expected rate that fixes the position of the Phillips curve that constrains its choices in the first year of the policy plan.

4.1   No anchoring ($\chi = 0$)

The case with no anchoring and a large inflation shock $\pi_0 > \pi^T$ presents the bank with some unpleasant arithmetic, since inflation is not self-stabilising and to disinflate requires a large sacrifice of output. The temptation for the central bank is to disregard the Hippocratic constraint and impose a sharp reduction in output in the first year. With zero anchoring, the effect on output according to our model is

$$y_1^{**} - y_{neut} = -\frac{\left(\pi_0 - \pi^T\right)}{\alpha}.$$

This loss would all occur in the first year. If the bank observes the Hippocratic constraint, the initial reduction in output will be less but the cumulative loss of output will be the same; this is the tyranny of the sacrifice ratio in action. Moreover, high inflation will persist for longer.

With no anchoring and no constraint on its behavior, the central bank problem has a straightforward two-stage solution:

$$y_1 - y_{neut} = -\frac{\left(\pi_0 - \pi^T\right)}{\alpha} \qquad \pi_1 = \pi^T$$

$$y_t = y_{neut} \qquad \pi_t = \pi^T \qquad t = 2, 3, \ldots.$$

The full sacrifice of employment takes place in one period, rather than being distributed across many. This generally does not happen in the standard (Carlin and Soskice, 2015) static decision problem underpinning the Taylor Rule because losses do not accumulate in that setting.

Things are just as bleak in the case of a pure demand shock or a mixed shock that puts the system in quadrant I or II in Figure 1. In either case, the solution has the same two-stage configuration. We could certainly explore the details of life in quadrants I and II but since the focus of the paper is the general case with anchoring we will move on. Purely inertial inflation is an historical vestige of the 1970s and 80s, but the last decades have seen dramatic changes in inflation behavior that includes stronger anchoring and our theorizing should take this into account.[17]

4.2   Anchored system ($0 < \chi < 1$)

In the presence of anchoring, the central bank problem undergoes a profound transformation because it is freed from the tyranny of the sacrifice ratio by the

---

[17]Ball and Mazumder (2011) estimate that the value of χ went from virtually zero before the 1990s to a range of 0.3 to 0.6 by 2010.

self-stabilizing nature of the inflation process.[18] In the case of weak anchoring, the singular path approaches π^T and $y_t = y_{neut}$ for large t. A central bank that *chooses* to use economic slack to speed up a disinflation will impose a loss of employment by design. The ratio of cumulative loss to the size of the disinflation is known as the sacrifice ratio. For a central bank that is unconstrained or ignores the constraints, the shortfall and sacrifice ratio are

$$
\begin{aligned}
\text{cumulative loss} \quad &= \quad \sum_{t=1}^{t=\infty} \left(y_{neut} - y_t \right) \\
&= \quad \frac{(1-\chi)\left(\pi_0 - \pi^T\right)}{\alpha} - \frac{\chi^2}{2\alpha^2 \beta b}
\end{aligned}
$$

$$
\begin{aligned}
\text{sacrifice ratio} &= \quad \frac{\sum_{t=1}^{t=\infty} \left(y_{neut} - y_t \right)}{\left(\pi_0 - \pi^T\right)} \\
&= \quad \frac{(1-\chi)}{\alpha} - \frac{\chi^2}{2\alpha^2 \beta b \left(\pi_0 - \pi^T\right)}.
\end{aligned}
$$

If $\pi_0 > \pi^T$ the sacrifice ratio is a decreasing function of χ. The more anchored are expectations, the lower is the cost of curbing inflation in terms of output foregone. Indeed, if χ is sufficiently large the sacrifice ratio will be negative. Tetlow (2022, Table 5) reports *ex post* ratios from simulations using the FRB/US model after a change in the inflation target, assuming the central bank follows a Taylor Rule. As we have argued here, the Taylor Rule has a disinflationary bias so it should come as no surprise that some sacrifice occurs from its application even under the most optimistic assumptions about the expectations formation process.

4.2.1   Pure inflation shock $\left(\pi_0 > \pi^T,\ y_0 = y_{neut}\right)$

In this case the solution is fairly obvious if the self-imposed constraints are binding. Since the Odyssean constraint prevents any significant expansion of output above potential, and the Hippocratic constraint limits the scope for using unemployment to combat inflation, the optimal policy is to keep output at the minimum or maximum level as appropriate until the singular path is reached. From then onwards, output and inflation converge geometrically to y_{neut} and π^T.

In the most rigid case, with $y_{low} = y_{neut}$ and $y_{high} = y_0$, a homeopathic policy of doing nothing[19] would be optimal. Of course, simply maintaining

---

[18]Formally, the equation of motion for expectations is

$$
\Delta \pi_t = -\chi(\pi_t - \pi^T)
$$

which is dynamically stable.

[19]Or nearly nothing since to keep the output gap closed the central bank would need to increase nominal interest rates in order to prevent the inflation shock from inadvertently delivering a stimulus from lower real interest rates.

output at its sustainable level would risk losing confidence since with a low coefficient of anchoring it may take considerable time for inflation to return to target. (At the same time, the steady decline in inflation would probably bolster confidence.) The intrinsic dynamic of inflation under a homeopathic policy is represented by a simple first-order difference equation so it is easy to calculate how long it will take to get within close reach of the target rate.[20] For a moderately high level of anchoring, say $\chi = 0.7$, after a 5% inflation shock it takes about two and a half years to come within 25 basis points of the target. For a low level, say $\chi = 0.3$ it takes over eight years, although inflation would only be one percentage point (100 basis points) above target after four and a half years.

[Figure 2 here.]

Figure 2 illustrates the impulse-response functions from the response to a pure inflation shock when there is a low degree of anchoring ($\chi = 0.2$). The economy is initially coasting along in steady state with $y = y_{neut}$ and $\pi = \pi^T$, when it is hit by a burst of high inflation. If the bank is unconstrained, given our choice of parameters it will respond aggressively, raising the interest rate sharply and provoking a steep fall in output. In our model, this eliminates the bulk of inflation within one period (a "year"). However, if the bank is constrained by the Hippocratic principle, it will pursue a less aggressive policy and seek to bring inflation down over the course of several periods.

Figure 2 illustrates the policy plan with an accrual factor that is larger than the critical value governing the sign of the output gap which is to say

$$b > \frac{\chi}{1 - \chi}.$$

This creates what might be described as the canonical policy dilemma. Respecting the self-imposed constraints requires a longer period of disinflation.

[Figure 3 here.]

However, this dilemma is not always salient in our model. Figure 3 illustrates a policy plan with an accrual factor that is less than its critical value. In this case, the central bank's unconstrained response calls for an immediate fall in output but then follows it up with a soft-landing with output slightly above its inflation-neutral level. The central bank is torn between conflicting goals but unlike Buridan's Ass it chooses to attend to one and then the other.[21] The constrained response then ends up invoking both the floor and ceiling we have chosen. The ceiling has been set equal to the inflation-neutral level on grounds that a central bank would face stiff resistance to any stimulative policy after a sharp inflation shock.

---

[20]The general solution for inflation with $y = y_{neut}$ is just $\pi_t = \pi^T + (\pi_0 - \pi^T)(1-\chi)^t$ so the time it takes to get within m basis points of the target is $(\log m - \log(\pi_0 - \pi^T))/\log(1 - \chi)$.

[21]The French philosopher Jean Buridan is eponymous for his paradox of a donkey who, placed equidistant between two piles of hay, starves because it cannot decide which to eat.

4.2.2 Demand shock ($\pi_0 > \pi^T$, $y_0 > y_{neut}$)

In the standard Taylor Rule framework, a positive demand shock is regarded as an adverse event requiring an immediate deflationary response that puts the system in quadrant II. In our framework, by contrast, there is no inherent need to rely on unemployment to solve inflation problems. Even without the Hippocratic constraint, a demand shock with or without an independent inflation shock can be resolved by means of a controlled disinflation which remains entirely in quadrant I. Figure 4 presents two such policy plans with different intrinsic weights (i.e., β) assigned to the inflation gap in order to illustrate how β affects the initial response. In both cases, the self-imposed constraints are non-binding.

[Figure 4 here.]

Once again, the shape of the policy plan depends critically on the choice of parameters. Putting less weight on the inflation gap could have invoked the Odyssean constraint at the front end of the plan. On the other hand, raising the penalty for persistent breaches could have eliminated the output gap more quickly, even taking the policy plan into quadrant II or perhaps hitting the Hippocratic constraint and relying on the self-adjusting property of inflation to finish the job of closing the inflation gap.

4.2.3 Stagflation ($\pi_0 > \pi^T$, $y_0 < y_{neut}$)

A negative demand shock in combination with a positive inflation shock (aka stagflation) calls for some restraint in restoring demand. How much restraint depends critically on the degree of inflation inertia, given fixed parameters in the loss function. The inflationary bias of the loss function potentially contributes to choosing a policy plan that exits quadrant II quickly. Figure 5 shows two examples that illustrate the role that the degree of anchoring plays in shaping the policy plan.

In this figure and the remaining figures we use a phase diagram environment as a canvas for visualising the policy plans which facilitates our references back to the four quadrants of Figure 1. A key to interpreting them is that the point labeled $t = 0$ represents the initial position, bullets and circles represent observations, and the lines connecting them represent trajectories that end at the destination with $y = y_{neut}$ and $\pi = \pi^T$.

In the first example, the parameters have been chosen so that the unconstrained policy creates a rapid escape from stagnation followed by a controlled return to potential output. (In this case, there may be some need to impose an upper limit on the stimulus but we have chosen to ignore the self-imposed constraints here.) In the second, a lower degree of anchoring prevents a rapid escape and requires a period of slack in order to bring down inflation owing to the greater degree of inertia. In this case, the policy plan adopts the profile of a traditional Taylor Rule trajectory.

[Figure 5 here.]

13

These policy plans highlight the difference between the socially responsible central bank and a bank deploying the standard Taylor Rule framework, which would have restricted the policy plan to quadrant II no matter what the degree of inertia unless it put no weight on the inflation gap. It is also interesting that the socially responsible bank will ignore the Taylor Principle in the low-inertia example since it will need to lower the real interest rate to achieve a recovery in demand, implying raising nominal interest rates by less than inflation has increased.[22]

5   Reflation

To complete the picture, consider shocks that lower inflation and call for a controlled reflation. In the event of a negative inflation shock that takes the system into quadrants III or IV, the same basic principles continue to apply. Here the standard Taylor Rule framework actually recommends an aggressive reflation that takes place entirely in quadrant IV. The difference is that our framework involves entering quadrant I and overshooting the inflation target.

5.1   Disinflationary shock $\left(\pi_0 < \pi^T,\ y_0 = y_{neut}\right)$

A disinflation shock without a rigid output ceiling will call for an immediate stimulus that enters quadrant I in the first period. Recall that the unconstrained optimal output level for period 1 is

$$y_1^{**} - y_{neut} = \frac{\chi}{2\alpha^2\beta(1+b)} - \frac{(1-\chi)}{\alpha}\left(\pi_0 - \pi^T\right).$$

A negative inflation gap requires an initially positive output gap to put the system onto the singular path. Moreover, if $b < \frac{\chi}{1-\chi}$, the output gap remains positive along the singular path. As we showed earlier, the inflation gap is always positive on the singular path implying overshooting the inflation target.

[Figure 6 here.]

There may be political obstacles to an ambitious reflation after a negative demand shock which prevent the system from reaching the singular path right away. Figure 6 illustrates the phase diagram for a constrained and unconstrained policy plan. In both cases, the reflation ends up overshooting the inflation target. In the constrained case, the output guardrail delays the onset of inflation overshooting but as long as the output ceiling permits a high-pressure labor market it will eventually occur. Only the extreme case of an output ceiling set at the inflation-neutral level of output prevents inflation overshooting in our model.

---

[22]The Taylor Principle is the general idea that the central bank needs to raise nominal rates by more than inflation has increased in order to deliver a disinflationary or contractionary response.

5.2 A mixed shock ($\pi_0 < \pi^T$, $y_0 > y_{neut}$)

To round out the discussion, consider an initial position that combines a negative inflation shock with a positive demand shock–the obverse of stagflation. It is difficult to think of historical examples of this felicitous state of affairs but we can use it to dramatize one of the properties of the model.

[Figure 7 here.]

Figure 7 illustrates a reflation with examples that have been parameterized to bring both self-imposed constraints into action. The Odyssean constraint can be motivated by resistance to additional stimulus since demand is already strong enough to speed the needed reflation. Here the output ceiling has been set at the initial (elevated) level of demand. In this example, the central bank maintains a high accrual factor that exceeds the critical value governing the sign of the output gap in periods beyond the first period (i.e., for $t > 1$). As a result the Hippocratic constraint remains relevant because of the Buridan's Ass problem we saw earlier and we have set the output floor at the inflation-neutral level. The central bank mechanically following its optimal program without constraints has been led to overindulge in stimulus and wind up paying for it by a period of unemployment later in the plan. In this case, both constraints will be activated. Figure 7 illustrates an example that exhibits (to use optimal control jargon) a bang-bang solution that alternates between the two self-imposed constraints.

6 Concluding comments

This paper challenges the standard approach to monetary policy that relies on a quadratic loss function on three grounds. First, it treats a high employment rate (i.e., above the inflation-neutral rate) as a social cost. The socially responsible central bank setting policy in this paper minimizes a loss function that recognizes high employment as a social benefit (and low employment as a cost). Second, the socially responsible bank observes an economic Hippocratic oath to do no harm by creating unnecessary unemployment.

Finally, the loss function in this paper recognizes that the costs of an inflation gap are time dependent. Inflation gaps after an initial shock are likely to erode confidence in the central bank's ability to achieve its inflation target the longer they persist and reduce the degree to which inflation expectations are anchored to the target. Because anchoring is a major social resource due to the flexibility it affords the monetary authority to stabilize inflation without imposing costs on workers, a socially responsible central bank will need to fashion any policy plan responding to a shock with its own credibility in mind. This is also a reason why incomes policies that reduce inflation inertia complement interest-rate policy.

With the global inflation shock generated by recovery from the Covid epidemic as a backdrop, we emphasize that the main difference in the policy plans between the current paper and the standard Taylor Rule approach is that the latter almost always involves the use of excess unemployment as a disinflationary

measure. A socially responsible central bank would only do that under extreme circumstances such as the absence of anchoring. It will generally try to take full advantage of the flexibility afforded by anchoring to operate the economy at or above its inflation-neutral employment rate, allowing inflation to stabilize through its own homeostatic tendencies. As of the date of writing, the U.S. experience seems to confirm the feasibility of this recommendation, although it remains possible that the Fed, following Taylor Rule protocol, has tightened sufficiently to induce a recession.

While the emphasis here is on managing disinflation, this basic difference also characterizes the optimal policy response to a negative inflation shock that requires reflation where the socially responsible bank will be willing to overshoot its inflation target.

Even though the central bank's decision problem is modeled using a precise mathematical technique (the Lagrangian method), it is clear from the examples and the discussion that central banking is as much an art as a science. In many cases, the central bank needs to make context-specific judgment calls about what constraints to impose on itself (including restraining its own inflationary biases), how to evaluate the cost of an inflation gap relative to an output gap, and how much to worry about the loss of credibility associated with persistence of the inflation gap. While it is often valuable to model behavior using optimization techniques, as in the present article, there is still a choice as to what is taken as given and what is endogenous. The framework offered here provides a disciplined environment in which to consider these issues. It is doubtful that monetary policy should be conducted with a simple interest rate reaction function like the Taylor Rule.

References

Ball, L. and Mazumder, S. (2011). Inflation dynamics and the Great Recession. *Brookings Papers on Economic Activity*, 41(1):337–405.

Barro, R. J. and Gordon, D. B. (1983a). A positive theory of monetary policy in a natural rate model. *Journal of Political Economy*, 91(4):589–610.

Barro, R. J. and Gordon, D. B. (1983b). Rules, discretion, and reputation in a model of monetary policy. *Journal of Monetary Economics*, 12(1):101–121.

Blanchard, O. (2016). The Phillips curve: Back to the '60s? *American Economic Review*, 106(5):31–34.

Blinder, A. S. (1998). *Central Banking in Theory and Practice*. MIT Press, Cambridge MA.

Brayton, F., Laubach, T., and Reifschneider, D. (2014). *Optimal-Control Monetary Policy in the FRB/US Model*. Board of Governors of the Federal Reserve System, Washington DC. FEDS Notes.

Carlin, W. and Soskice, D. (2015). *Macroeconomics: Institutions, Instability and the Financial System*. Oxford University Press, Oxford.

Carlin, W. and Soskice, D. (2018). Stagnant productivity and low unemployment: Stuck in a Keynesian equilibrium. *Oxford Review of Economic Policy*, 34(1-2):169–194.

Dornbusch, R. (1976). Expectations and exchange rate dynamics. *Journal of Political Economy*, 84(6):1161–1176.

Foley, D. K., Michl, T. R., and Tavani, D. (2019). *Growth and Distribution, Second Edition*. Harvard University Press, Cambridge MA.

Mankiw, N. G. (2006). The macroeconomist as scientist and engineer. *Journal of Economic Perspectives*, 20(4):29–46.

Michl, T. R. (2018). Hysteresis in a three-equation model. *Eastern Economic Journal*, 44(2):305–322.

Michl, T. R. and Oliver, K. M. (2019). Combating hysteresis with output targeting. *Review of Keynesian Economics*, 7(1):6–27.

Oreiro, J. L. and Santos, J. F. C. (2023). The unfinished stabilization of the *Real Plan*: An analysis of the indexation of the Brazilian economy. In Filho, F. F. and de Paula, L. F., editors, *Central Banks and Monetary Regimes in Emerging Countries: Theoretical and Empirical Analysis of Latin America*, pages 61–81. Edward Elgar, Cheltenham.

Taylor, J. B. (1993). *Discretion Versus Policy Rules in Practice.* Carnegie-Rochester Conference Series on Public Policy, Washington DC.

Taylor, L. (2004). *Reconstructing Macroeconomics: Structuralist Proposals and Critiques of the Mainstream.* Harvard University Press, Cambridge, MA.

Tetlow, R. J. (2022). *How Large is the Output Cost of Disinflation.* Federal Reserve Board, Washington DC. Finance and Economics Discussion Series 2022-079.

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton University Press, Princeton.

Yellen, J. (2012). *Perspectives on Monetary Policy.* Board of Governors of the Federal Reserve System, Washington DC. Speech to the Boston Economic Club.

$$(\pi - \pi^T)$$

II	I
$\pi > \pi^T \quad y < y_{neut}$	$\pi > \pi^T \quad y \geq y_{neut}$

$(y - y_{neut})$

III	IV
$\pi < \pi^T \quad y \leq y_{neut}$	$\pi < \pi^T \quad y > y_{neut}$
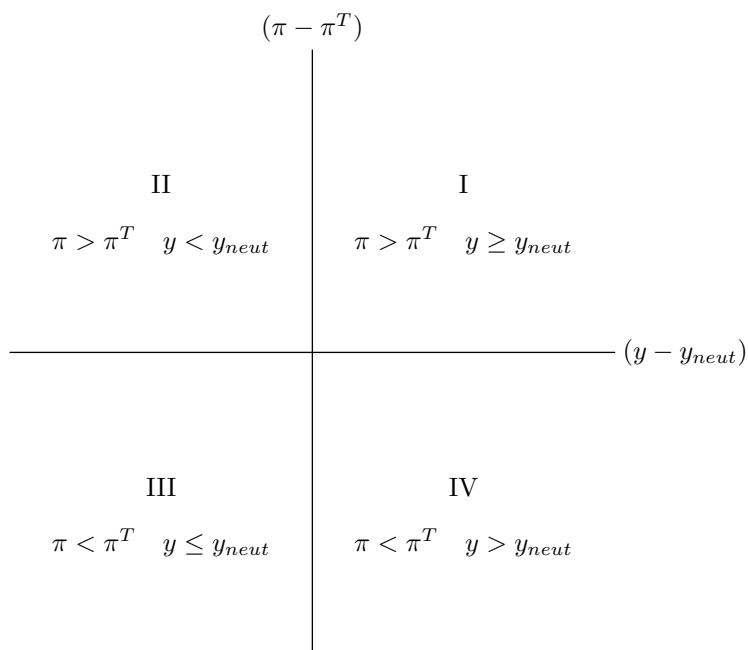
Figure 1: Policy zones defined by output and inflation gaps. The standard Taylor Rule restricts the policy plan to quadrants II and IV. The alternative loss function recommends policies that make use of quadrant I, and discourages use of quadrant II.
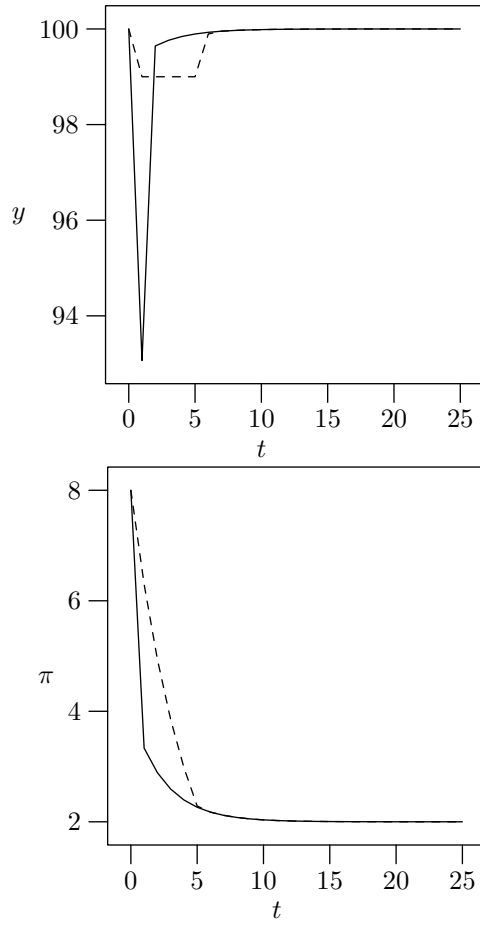
Figure 2: Impulse-response functions for a pure inflation shock (+6) with (dashed line) and without (solid line) self-imposed constraints. A high accrual factor creates a difficult decision for the central bank since its constrained choice risks losing credibility. The parameters are $y_{neut} = 100$, $y_{low} = 99$, $y_{high} = 100$, $\pi^T = 2$, $\chi = 0.2$, $\alpha = 0.5$, $\beta = 0.5$, $b = 0.5$.
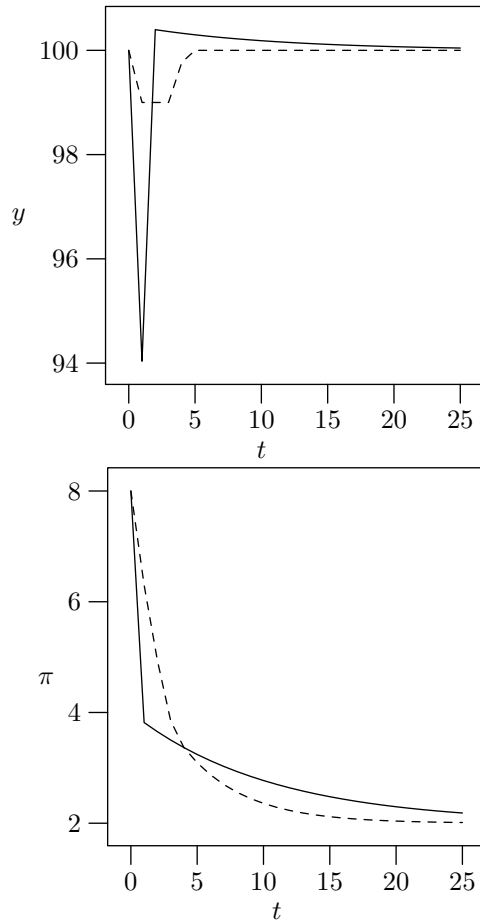
Figure 3: Impulse-response functions for a pure inflation shock (+6) with (dashed line) and without (solid line) self-imposed constraints. A low accrual factor brings both constraints into effect. The parameters are $y_{neut} = 100$, $y_{low} = 99$, $y_{high} = 100$, $\pi^T = 2$, $\chi = 0.2$, $\alpha = 0.5$, $\beta = 0.5$, $b = 0.1$.
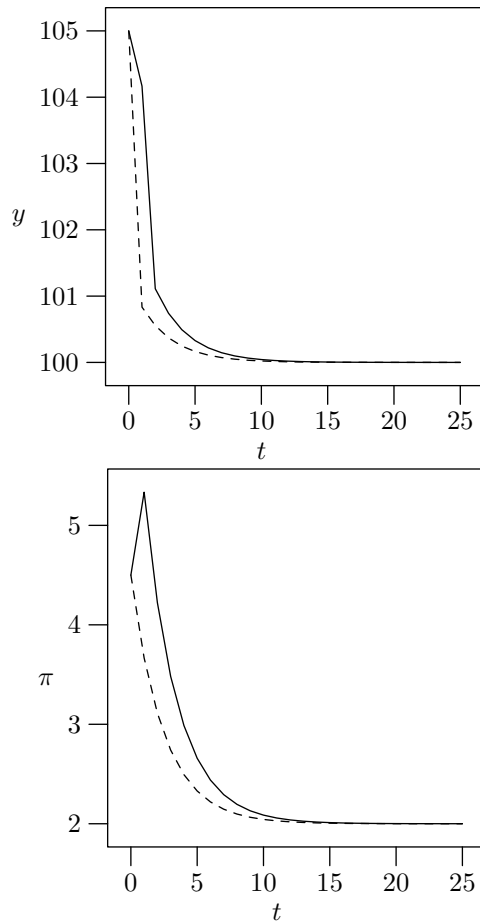
21

Figure 4: Impulse-response functions for a demand shock (+5) with $\beta = 0.1$ (solid line) and $\beta = 0.2$ (dashed line). Both plans stay within quadrant I (defined with respect to output and inflation gaps). The parameters are $y_{neut} = 100$, $\pi^T = 2$, $\alpha = 0.5$, $\chi = 0.5$, $b = 0.5$.
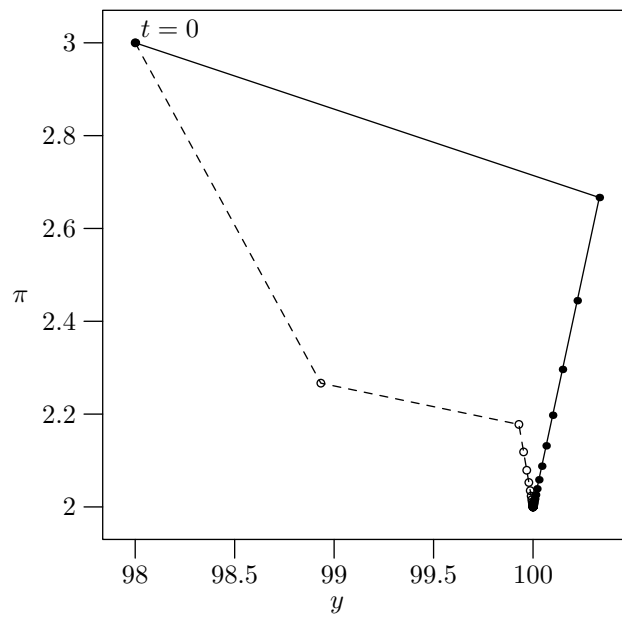
Figure 5: Phase diagram for stagflation from a demand shock (-2) and an inflation shock (+2) with $\chi = 0.2$ (dashed line, circles) and $\chi = 0.5$ (solid line, bullets). A lower degree of inflation inertia (higher χ) permits the central bank to recover from the shock through quadrant I. The parameters are $y_{neut} = 100$, $\pi^T = 2$, $\alpha = 0.5$, $b = 0.5$, $\beta = 0.5$.
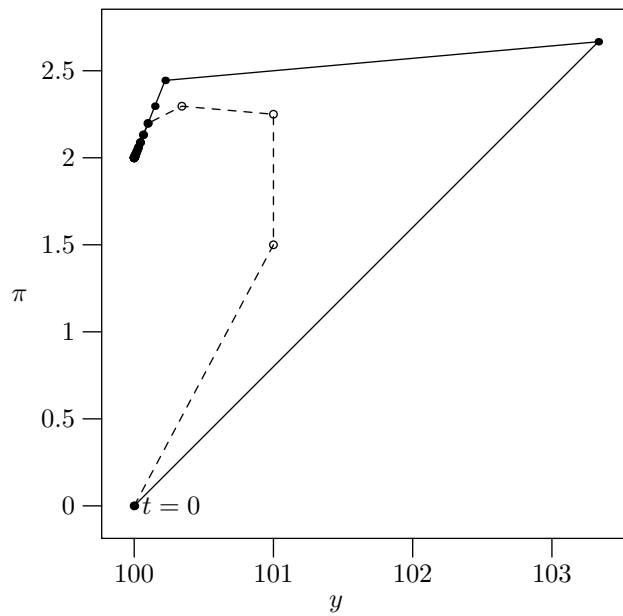
Figure 6: Phase diagram for reflation after a negative (-2) inflation shock with (dashed line, circles) and without (solid line, bullets) a self-imposed output ceiling. The parameters are $y_{neut} = 100$, $y_{low} = 100$, $y_{high} = 101$ $\pi^T = 2$, $\alpha = 0.5$, $\chi = 0.5$, $\beta = 0.5$, $b = 0.5$.
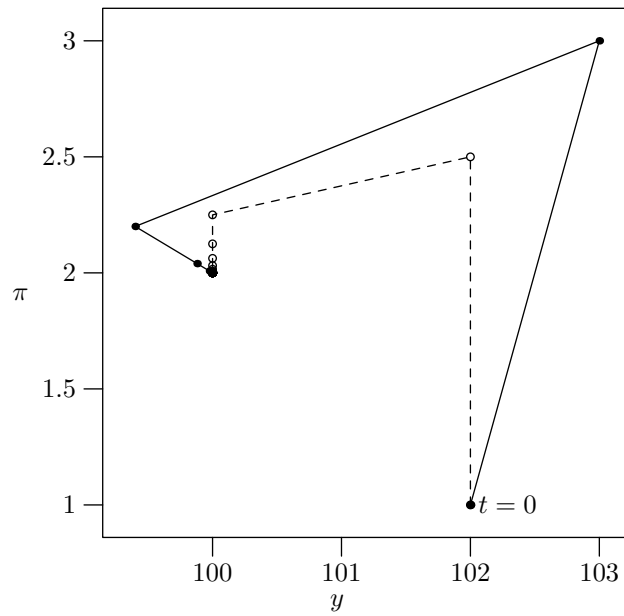
24

Figure 7: Phase diagram for reflation with a positive (+2) demand shock and negative (-2) inflation shock with (dashed line, circles) and without (solid line, bullets) constraints. The constraints produce a bang-bang solution. The parameters are $y_{neut} = 100$, $y_{low} = 100$, $y_{high} = 102$ $\pi^T = 2$, $\alpha = 0.5$, $\chi = 0.5$, $\beta = 0.1$, $b = 4$.