



# **A Three Class Predator-Prey Model with Financial Super Predators: The Financial Profit Squeeze**

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## I. Introduction

Numerous heterodox analyses of the Great Recession exist.<sup>1</sup> While most contributions integrate the role of financial capitalists, their role is typically limited to the creation of asset bubbles and the resulting financial crises. The integration of financial capitalists is rarely grounded in adversarial class relations, but alternatively in the liberalization of the financial sector. In contrast, this paper considers the nexus of coercive power relations between labor (L), industrial capitalists (IK) and financial capitalists (FK) and their impact on dynamic macroeconomic performance in the Neo-liberal (NL) era.

In particular, a finance capital-dominated profit-led accumulation regime is modelled in which FK are primarily responsible for a squeeze of IK profits<sup>2</sup> and secondarily for reductions in labor's share of income and unintended asset bubbles that act as a temporary countervailing tendency to a FK profit squeeze (PS) and under-consumption crisis. These dynamics are captured by extending Goodwin's (1967) two class predator-prey model to include the impact of a three-way class/power struggle over the distribution of income.

The primary purpose of this paper is to: 1) introduce a three-class predator-prey model with class interactions consistent with the NL era into the literature; 2) analyze the spectrum of

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<sup>1</sup> For example, see Kotz (2013, 2015), Glyn(2007), Crotty(2003), Turner(2008) and Foster and McChesney (2012).

<sup>2</sup> This squeeze of profits is qualitatively distinct from a reserve army profit squeeze but is not necessarily mutual exclusive of such a profit squeeze.

different qualitative macroeconomic behaviors associated with the model; 3) focus on two particular outcomes – FK PS and a double reserve army (RA) – FK PS –both capable of producing cyclical and chaotic outcomes; and 4) use the FK PS to explain key stylized facts associated with the NL period. Given a sparing behavioral structure for the model where financial markets are not explicitly specified<sup>3</sup>, a full-on calibration exercise is not attempted.

A well-developed literature on the historical succession of regimes of accumulation exists. A subset of that literature (Bowles et al. (1990), Goldstein (2000), Gutierrez-Bargarrusa (2019; p. 1307) Jaffee (2019; pp.193-6), and Weisskopf et al. (1985)) cite evidence of cycles between alternative accumulation regimes that underlie the difficulty in achieving balanced growth/expansion within capitalism. A succession of a profit-led (unregulated) regime supplanted by a wage-led (regulated) regime with their respective crisis-producing mechanisms of under-consumption and RA PS crises highlight the difficulty in achieving long-term balanced growth. Each regime ends as a result of an unbalanced increase in the power of one of the main productive classes. Yet, however inefficient, in the long-run the conditions for profitable accumulation are re-established by a transition between regimes.

The fundamental distinction in a FK-dominated profit-led regime is that the crisis combines the worst of both alternative regimes. Growth is brought to a standstill by a FK PS that lowers investment and by an under-consumption crisis when deficiencies in labor's income can no longer be overcome by debt accumulation.

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<sup>3</sup> Only the redistribution of income from L and IK to FK is considered, but a linkage is made from FK income to increases in wealth via an implicit channel. It is assumed that FK income used to trade on their own accounts increases asset prices and thus wealth.

The most popular extension of two species predator-prey models includes a super-predator along with a predator and a prey. The super-predator preys on both other species and faces no predation (regulation). In this paper, FK is the super-predator. In general, crises are endemic to such models. The unregulated coercive competitive forces operating among super-predators can lead to the excessive depopulation of other classes. While this behavior has long been recognized in the biology/ecology literature, I refer to it as the super-predator paradox or in our case, the FK paradox. This is especially problematic when the super-predator is primarily an unproductive element of society. As a result, one possible outcome is a long period of stagnant growth. Attempts to strengthen one of the productive classes without reducing the power of the super-predator, can result in more destabilizing behavior. For example, progressive attempts to strengthen L without weakening FK, can result in a double RA- FK PS with short-lived chaotic economic performance.<sup>4</sup>

Our simulated results show that a three-class model substantially expands the possible accumulation regimes and crisis mechanisms that can be analyzed. Both wage-led, profit-led and modified regimes such as a FK dominated profit-led regime can be studied. In addition, under-consumption, FK PS, RA PS and a double FK-RA PS can impede accumulation/growth. Thus, the three class model is versatile.

The paper focuses on the historically relevant FK PS and FK-RA PS and reveals that, under relevant parameter values, cyclically volatile behavior in the form of transient chaos, limit cycles and long-lasting damped cycles are likely outcomes. In general, the volatile disruption of

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<sup>4</sup> The short-term nature of the chaos is associated with a destabilized limit cycle that experiences turbulence/bursts as a result of a combination of a fundamental fast cycle and a sub-harmonic slow cycle.

high employment equilibriums or turbulent depressed equilibriums or in a growth context (not considered) the hinderance of balanced growth are associated with severe imbalances in power relations across the three classes. Of particular relevance is that substantial increases in FK power have major negative consequences on macro performance.

The remainder of this paper is organized as follows. Section II presents the salient stylized macroeconomic facts associated with the NL era. Section III develops the differential equation model and the supporting expenditure-income (employment) model along with the simplifying assumptions invoked. A base unbounded three equation/class DEQ model is specified followed by a four-equation model with upper and lower bounding of state variables. The latter incorporates an additional macroeconomic state variable, wealth, as a means of placing a lower bound on expenditures and hence employment. Section IV addresses the qualitative behavior associated with the unbounded model based on simulated solutions. Section V presents the qualitative results from the bounded model with emphasis on the FK-RA PS and the FK PS. Section VI reports the results of sensitivity analysis and Section VII contains concluding comments.

## II. Stylized Facts

Analysts of the US economy in the NL era focus on various stylized facts. The type of variables, considered are: 1) indices of shifts in power relations between L, IK and FK; 2) profit rate decompositions including labor/profit share of income, technical ratios and capacity utilization; 3) the intensification of international competition; 4) the composition of expenditures as shares of aggregate demand; and 5) private debt and wealth.

Relevant to the model developed, the salient facts are: 1) the weakening of L vis a vis IK; 2) the absolute and relative rise to power of FK with respect to IK; 3) the unexpected, for a profit-led regime, rise (decline) in consumption (investment) as a share of aggregate demand; and 4) increases in private debt facilitated by increases in wealth, via asset bubbles, that underlie the rise in the consumption share.

Given that 1) is well documented, only the resultant secular decline in the labor share of income is considered. The shift in power between FK and IK is demonstrated through a comparison of the IK gross profit share of income and the profit share net of payments to FK.<sup>5</sup> In addition, the trend in consumption share is shown along with the trajectory of real wealth<sup>6</sup>. Finally, the ratio of consumption to investment which tracks the relative change in each share of GDP is reported. The data from 1980-2012 are summarized in Figure 1, respectively in panels a-d. Variable definitions and data sources are contained in Appendix A.

In Figure 1, panel A shows the well-documented secular decline in the labor share of income ( $\mu$ ).<sup>7</sup> From 1980-2015,  $\mu$  declines from .58 to .52. Typical cyclical fluctuations are also present. Cyclical increases in  $\mu$  are considered, for the most part, to be weak so as not to support a cyclical RA PS (Goldstein (1999) and Boddy and Crotty (2018)). Although, Basu et al. (2013) and Boddy and Crotty (2018) conclude that the cyclical RA PS is operational in at least one of

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<sup>5</sup> Bezrah and Goldstein (2013) use a liberal, but defensible, definition of the IK profit share that is net of net dividends, net stock purchases, and net interest payments.

<sup>6</sup> Debt is not explicitly considered in the model but is rather captured by increases in wealth that allow for expanded debt limits.

<sup>7</sup> Alternative measures of labor share show the same trend. See Giandrea and Sprague (2017).

the NL cycles.<sup>8</sup> This finding is relevant because it lends credence to the practical possibility of a double FK-RA PS that is addressed in our model.

Panel B displays the IK profit share of income prior to and net of net interest, dividends and stock purchases. The recovery of the gross profit share from its decline during the latter stages of the Golden Age is evident. This is consistent with a profit-led accumulation regime. While numerous authors have emphasized the rise in FK income<sup>9</sup>, none have calculated its impact on the IK profit share. The net share accomplishes this. The widening gap between the gross and net shares in panel B and the downward trend in the net share capture important aspects of the absolute rise to power of FK and more importantly, the relative power shift between IK and FK.<sup>10</sup>

Thus, net of coercive transfers of profit, the IK profit share did not recover and has continued to decline. In other words, FK captured more than the total gains made from increases in the rate of labor exploitation. The gap between the two graphs represents FK share of income (f). This upward trend in f is what I refer to as the FK PS.<sup>11</sup>

Panel C shows the share of GDP going to consumption  $c$  and real wealth in trillions of dollars. The well-documented increase in  $c$  is substantiated. Given the damping effect of a

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<sup>8</sup> Basu et al. (2013) find that the RA PS is generally operable throughout the period. Boddy and Crotty (2018) argue that a RA PS occurs in the 1990s cycle. They are unable to determine if a PS occurs during the 2001-2007 cycle due to the start of the financial crisis.

<sup>9</sup> See Crotty (2003, 2008), Turner (2008) and Glyn (2007), Kotz (2015) and Foster and McChesney (2012).

<sup>10</sup> Numerous papers by Crotty (2003, 2008, 2009) clearly express the adversarial/coercive nature of these transfers. Analyses of the profit rate that do not consider the impact of transfers, such as in Kotz (2013), are likely to underestimate the role of profitability problems as a cause of the Great Recession. Oftentimes this oversight is associated with the view that such transfers are voluntary/non-coercive.

<sup>11</sup> In contrast, Bezrah and Goldstein (2013) show that  $f$  remains basically constant during the regulated capitalism regime.



decline in IK net profit share on I and FK innovation and speculative trading responsible for asset bubble-induced wealth effects on C, c has risen dramatically. The wealth data pinpoints three asset bubbles (mid-late 1980s and mid 2000s housing price bubbles and the late 1990s Dot Com bubble) that correspond with more rapid increases in c. These bubbles also correlate well with increases in f.

Finally, panel D depicts the ratio of C to investment expenditures (I) as the C-I ratio. The relative behavior of the I share in the NL era is the least agreed upon fact among researchers. Some authors (Kotz (2013, 2015) and Turner(2008)) analyze an over-I crisis, while others favor either under-I (Glyn(2007)) or a mixture depending on the composition of industries considered.<sup>12</sup> The C-I ratio increases from 1980-1991 and 2001-2006 and declines from 1992-2000. The rise corresponds with increases in f and the occurrence of asset bubbles. The decline is related to the long I-led expansion of the 1990s that includes the Dot-Com bubble, but also a decline in f that created a more conducive I environment.

The relative rise in C share prior to the Great Recession is indicative of wealth-induced increases in c as an offsetting crisis tendency in an environment with relatively damped I activity.

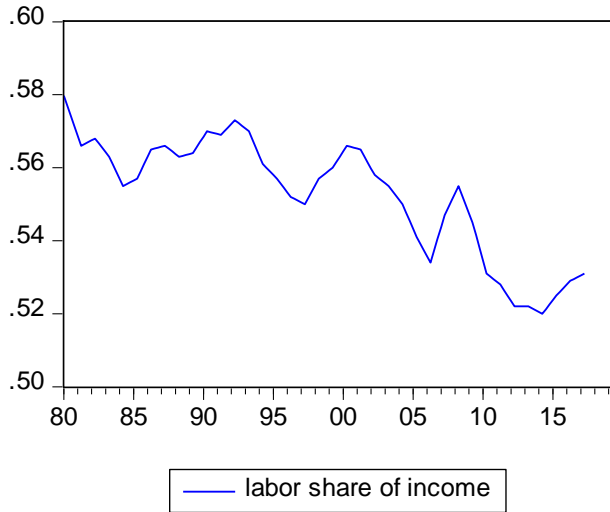
Given the salient stylized facts, we turn to a model capable of explaining these outcomes.

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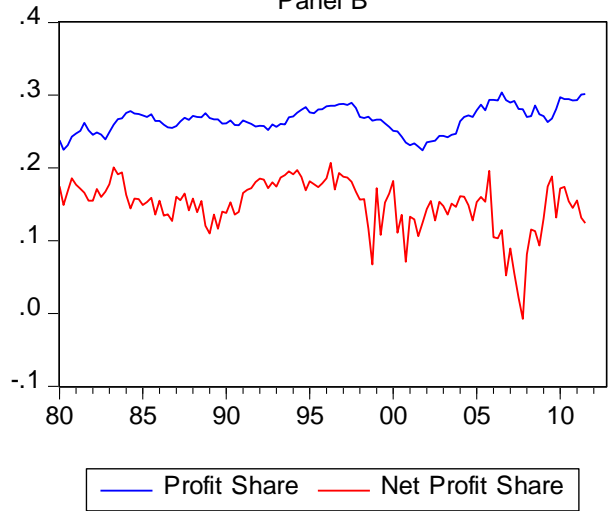
<sup>12</sup> Crotty's (2003) analysis of the NL regime considers global excess capacity created by international competition-induced defensive I spending. Crotty and Goldstein (1992a, 1992b) and Goldstein (2009) develop a model of the firm's I decision that has increasing I as competition rises and decreasing I with respect to transfers of profit to FK increases.

Figure 1. Stylized Facts

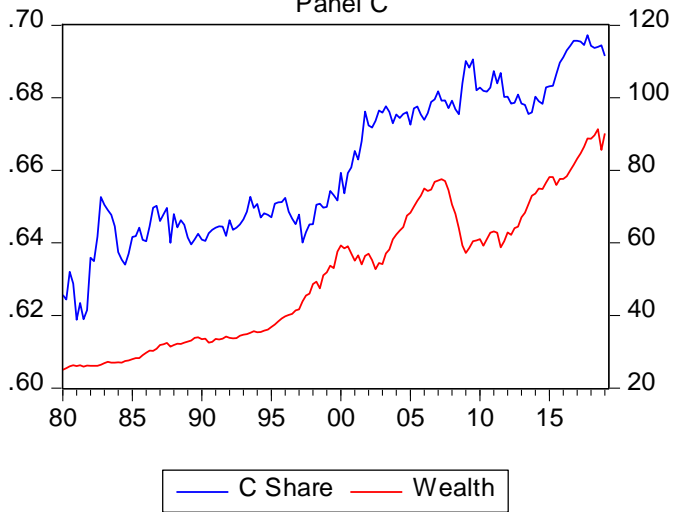
Panel A



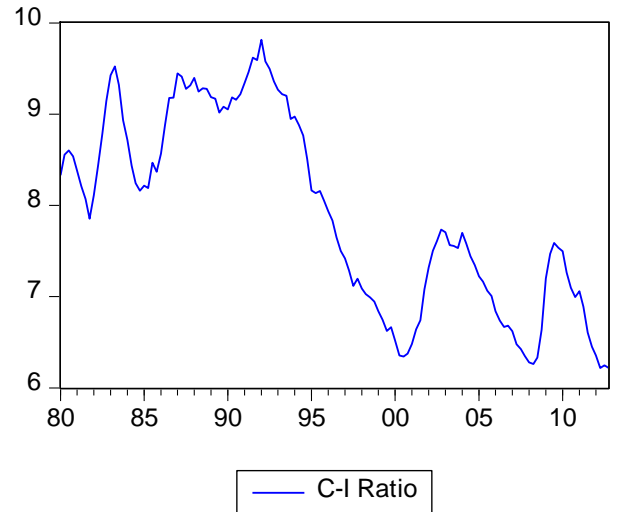
Panel B



Panel C



Panel D



### III. Predator-Prey Models

This section develops a base three class predator-prey model and also an extended/bounded model. The extended model includes a fourth DEQ.

Goodwin's two class- two differential equation (DEQ) model includes an equation for the relative power shifts between IK and L. The second equation captures changes in a macro state variable, the degree of full employment ( $v$ ), in response to the relative power between classes.

The three class/three DEQ model contains two equations for relative shifts in the balance of power between three classes (IK, L, FK) and a third for the same macro state variable. One equation models how power relations between all three classes affect labor share of income ( $\mu$ ), while the other considers how IK-FK relations impacts FK share of income ( $f$ ) and hence the net IK share,  $(1-\mu-f)$ . The equation of motion for  $v$ , is determined by imbalances in a short-term expenditure-income framework where expenditures depend on the state variables rather than from a growth framework.<sup>13</sup> In the model below, FK is the super predator, while IK is the predator and L the prey.<sup>14</sup>

In contrast to Goodwin's model, the current model is more flexible. This occurs on two levels: 1) the qualitative behavior of state variable trajectories; and 2) the nature of class interactions. With respect to the former, Goodwin's model is confined to limit cycle behavior and for the latter, L is viewed as the predator of profits. Since three DEQs are a necessary

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<sup>13</sup> Tobin (1975) marries an E=Y model within a DEQ model to derive stability conditions for Keynesian short-run dynamics.

<sup>14</sup> The roles of IK and L can easily be reversed on the basis of different parameter values. The analysis below considers both the case of strong and weak labor vis a vis IK.

condition for chaos, chaotic results are possible in our model assuming that dynamics are sufficiently bounded globally.<sup>15</sup> Additionally, limit cycles and damped cycles are possible.

On the second level, Table 1 summarizes different types of class interactions and resulting behaviors possible in the model. In the table, the > symbol refers to the dominance (predation) of one class over (by) another. The table only considers cases where FK dominates L (FK>L). A variety of class dynamics and outcomes are feasible.

Table 1. Power Relations and Possible Outcomes

CLASS RELATIONS	POSSIBLE OUTCOMES
IK>L, IK>FK and FK>L	Profit-led growth with FK as servant and possible Under-consumption crisis
IK>L, FK>IK and FK>L	FK dominated profit-led regime with FK profit squeeze and possible under-consumption crisis
L>IK, FK>IK and FK>L	Wage-led growth with a double RA-FK squeeze of profits
L>IK, IK>FK and FK>L	Wage-led growth with a RA profit squeeze

In the analysis below, the likelihood of under-consumption problems are minimized by incorporating household borrowing to sustain consumption and wealth-induced consumption as

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<sup>15</sup> Goodwin (1991) adds an ad hoc dynamic control variable in the form of a DEQ for government policy and demonstrates that chaotic behavior can result. Here, chaos can result more naturally from the endogenous interactions between classes.

offsetting tendencies. It is not until a significant decline in wealth occurs ( via a pseudo bubble burst)<sup>16</sup> that under-consumption appears.

With respect to boundedness and stability, the model developed does not readily fit the typology of three-species predator-prey models for which analytical results have been established.<sup>17</sup> Thus, general results are not available. Results of interest are generated from numerical simulations.

#### a. Underlying Model Assumptions

For simplicity, FK are integrated as unproductive economic agents. All FK income are transfers from either profit or wages or both. While the model is amenable to treating both FK fiduciary and intermediary roles, interest income is not explicitly modelled. Thus, FK income is transferred from IK in the form of stock buybacks and dividend payouts. The implications for income shares are that  $\mu + (1-\mu)$  (labor share plus the gross IK profit share) exhaust total income and  $\mu + (1-\mu-f) + f = 1$ . FK income is divided between limited FK consumption and trades made on FK accounts.<sup>18</sup> Given the large percentage of FK income devoted to speculative trading,  $f$  is linked to wealth via presumed asset demand-induced price increases.

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<sup>16</sup> Given the difficulties of integrating a discontinuous bubble collapse into a continuous model, only continuous cycles in wealth are modelled.

<sup>17</sup> Krikorian (1979) develops general boundedness and stability conditions for thirty-four variants of the three-species model including variants based on particular parameter ranges.

<sup>18</sup> The unproductiveness of FK activity, implies that all such trades are made in secondary markets.

Taking unemployment and excess capacity as typical, we make production/supply-side assumptions to establish a linkage between demand-induced increases in supply ( $Y$ ) and the degree of full employment,  $v$ .<sup>19</sup> Assuming fixed coefficients, constant returns to scale production, a given percent increase in supply will increase  $v$  by the same percentage. Thus  $\frac{\dot{v}}{v} = \frac{\dot{Y}}{Y}$  where variables with dots are time derivatives.

Further assumptions are addressed in the model presentation.

The formal model consists of three DEQs for  $\mu$ ,  $f$ , and  $v$  and an associated income-expenditure model.

#### b. Three Class Predator-Prey Model

An equation of motion for labor share of income,  $\mu$ , is as in Goodwin (1967) with the addition of a third term capturing the impact of FK on  $\mu$ .

$$\frac{\dot{\mu}}{\mu} = -\Upsilon + \rho_1 v - \rho_2 f \quad (1)$$

where  $v$  is the degree of full employment,  $f$  is FK share of income and  $\Upsilon$ ,  $\rho_1$  and  $\rho_2$  are positive constants. Equation (1) captures the conflict between IK and FK vs. L over  $\mu$ .  $\Upsilon$  is the negative trend in  $\mu$  from secular declines in trade union density, labor market deregulation and the weakening of labor legislation to mention a few.  $\rho_1 v$  considers shifts in the IK-L balance of

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<sup>19</sup> It should be noted that in a short-run model with fixed labor supply and constant productivity, the only determinant of  $\frac{\dot{v}}{v}$  is the percentage change in employment.

power associated with the bargaining power of labor determined by the RA. Boddy and Crotty (2018) and Goldstein (1999) empirically establish that this effect has been significantly weakened during the NL era. Combinations of  $\Upsilon$  and  $\rho_1$  model a continuum of the relative power of L vs. IK. In addition, weakened cyclical effects ( $\rho_1$  small) allow for a more secular focus.<sup>20</sup>

The relative strength of L vs. FK is in  $\rho_2 f$ . As  $f$  increases, the political and economic sway of FK increases. This facilitates FK occupation of corporate board positions, influence over corporate policy, implementation of cost rationalization strategies (downsizing, use of contingent labor, etc.) and financing of capital flight all of which impact  $\mu$ .<sup>21</sup> More importantly,  $\rho_2 f$  indirectly models the IK response to declining net profits through low road labor strategies.

Equation (2) models the interactions between IK and FK primarily through the shareholder value channel.

$$\frac{\dot{f}}{f} = -\alpha_0 + \alpha_1(1-\mu) + \alpha_2 v \quad (2)$$

where  $\alpha_0$ ,  $\alpha_1$ , and  $\alpha_2$  are positive constants.  $\alpha_0$  includes exogenous influences on  $f$  such as financial regulation/deregulation that limit IK transfers unrelated to the level of profits. The  $1-\mu$  and  $v$  terms represent profit available for transfer broken respectively into per-unit profitability and the scale of profitable units. More FK influence/power, as measured by  $f$ , translates to larger profit claims in the form of coerced higher dividends, stock buybacks and monopoly pricing on

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<sup>20</sup> Given asymmetries in regaining (relinquishing) power after secular declines (gains), a more appropriate specification could include an asymmetric response where  $\rho_1$  takes on different values for  $\dot{v} > 0$ . This extension is beyond the scope of this paper.

<sup>21</sup> For a summary of the theoretical FK and NL channels for the secular decline in  $\mu$ , see Barradas (2019).

financial innovations supplied to IK. In this base model, FK claims on IK profits are not limited. In the bounded model, they are restricted.

Adjustments to the macroeconomic state,  $v$ , are determined by imbalances between income share-determined aggregate demand and perfectly elastic aggregate supply. Thus,

$$\frac{\dot{v}}{v} = \frac{\dot{Y}}{Y} = A(\mu, f) [c(\mu, f) + i(\mu, f) + g - 1] \quad (3)$$

where  $Y$  is the level of production/income,  $A$  is the Keynesian multiplier and  $c$ ,  $i$ , and  $g$  are respectively the consumption, investment and government expenditure shares of  $Y$  ( $\frac{c}{y}$ ,  $\frac{I}{Y}$ , and  $\frac{g}{y}$ ).<sup>22</sup>  $A$  is endogenously determined by income shares. In the simple model without bounding,  $G$  policy is excluded ( $G = 0$ )).

### c. The Income-Expenditure Model

The expenditure model includes a class-based Keynesian consumption function and a Marxian equation for the accumulation of capital. A three-class consumption function takes the following form,

$$C = \beta_0 + \beta_L \mu Y + \beta_{IK} (1 - \mu - f) Y + \beta_{FK} f Y$$

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<sup>22</sup> A more familiar form of equation (3) is derived from  $\dot{Y} = A(\mu, f) [E(\mu, f) - Y]$  and  $\frac{\dot{Y}}{Y} = A(\mu, f) [E(\mu, f)/Y - 1]$ . Under our production function assumptions, the existence of excess capacity and  $E = C + I + G$ ,  $\frac{\dot{v}}{v} = \frac{\dot{Y}}{Y} = \text{RHS EQ (3)}$ .



where all  $\beta$  are positive constants. Under the simplifying assumption that all profits are invested ( $\beta_{IK} = 0$ ), the economy-wide marginal propensity to consume is  $\beta_L \mu + \beta_{FK} f$ . The class-based C function is modified to include both L attempts to preserve its historical level of C through borrowing and an economy-wide wealth effect. Thus,

$$C = \beta_0 + \beta_L \mu Y + \beta_{FK} f Y + \beta_1 (\mu^* - \mu) Y + \beta_2 W \quad (4)$$

where  $\mu^*$  is a socially and historically acceptable L share,  $(\mu^* - \mu)$  is the share deficit, W is real wealth and  $\beta_1$  and  $\beta_2$  are constants.

Share deficit C spending is assumed to be debt financed. Interest payments on this debt are not considered, but the impact of debt on net worth (W) is considered in the extended model. In the base model, the wealth effect is not considered ( $\beta_2 = 0$ ).

The share deficit term, neutralizes tendencies to under-consumption from declining  $\mu$  making it more likely that a profit-led regime exists.<sup>23</sup> Thus, reductions in wealth are the primary channel for C to decline.

A Marxian equation for the accumulation of capital is

$$\frac{I}{K} = Z_1 (1 - \mu - f) \frac{Y}{K}$$

where K is the capital stock,  $Z_1$  is a constant often set equal to 1. Thus, the rate of accumulation equals the net profit rate. Under the assumption of a constant  $\frac{K}{Y}$  ratio,

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<sup>23</sup> When  $\beta_L = \beta_1$  and  $\mu$  increases or decreases by a unit, the marginal impact on C is zero. Even when  $\beta_L$  and  $\beta_1$  are not equal, the marginal effect is significantly less than if  $\beta_1 = 0$ . For reasonable parameter values  $\partial Y / \partial \mu < 0$  (condition for an I-led regime).

$$\frac{I}{Y} \frac{Y}{K} = \frac{I}{K} Z_1 (1 - \mu - f) \frac{Y}{K} \text{ and } I = Z_1 (1 - \mu - f) Y$$

Modifying I to include exogenous influences and a potential wealth effect,<sup>24</sup>

$$I = Z_0 + Z_1 (1 - \mu - f) Y + Z_2 W \quad (5)$$

where  $Z_0$  and  $Z_2$  are positive constants.

Fiscal policy, not integrated in the base model, is based on closing recessionary/inflationary gaps,

$$G = (v^* - v) \frac{Y_f}{A^e} \quad (6)$$

where  $v^*$  is the target level of  $v$ ,  $Y_F$  the full employment level of  $Y$ ,  $v Y_F = Y$  and  $A^e$  is an expectation of the endogenous multiplier.

Equations (4-6) when divided on both sides by  $Y$  to generate  $c$ ,  $i$  and  $g$  are substituted into equation (3) to complete the three DEQ model.<sup>25</sup>

The base model is constituted by equations (1-2) and equation (3) with  $c$ ,  $i$  and  $g$  substituted in with the following restrictions:  $g=0$ ,  $\beta_{IK} = 0$ ,  $Z_1 = 1$  and  $\beta_2 = Z_2 = 0$ .

Parameters that determine relative class positions are:  $Y$ ,  $\rho_1$ ,  $\rho_2$  directly and  $\rho_1$  vs.  $\alpha_2$  indirectly for IK vs. L,  $\rho_2$  directly and  $\alpha_1$ ,  $\alpha_0$  and  $\rho_1$  vs.  $\alpha_2$  indirectly for FK vs. L and  $\alpha_0$ -  $\alpha_3$  directly and  $\rho_1$  vs.  $\alpha_2$  indirectly for IK vs. FK. In addition,  $W_0$  and  $W_1$  (introduced below) affect

<sup>24</sup> In simulations, the possibility that  $I$  falls below depreciation  $I$  should be considered. Fortunately, in our simulations, it was not necessary to impose this floor on  $I$ .

<sup>25</sup> Note that  $\frac{Y_f}{Y} = \frac{1}{v}$  and when  $g \neq 0$ , that  $g = g(v)$  and thus,  $A = A(u, f, v)$ .

the absolute position of FK. When initial conditions are considered,  $f(0)$  affects IK vs. L and FK vs. IK, while  $\mu(0)$  impacts IK vs. FK and FK vs. IK via the interactions terms in the time derivative variant of the three equations.

While the basic model suppresses under-consumption tendencies ( $\beta_2 \neq 0$ ), it captures various profit squeezes (RA, FK and FK-RA). The underlying dynamics of the FK PS outcome are considered. Starting from a secular decline in  $\mu$ , IK gross profits rise. This redistribution has minimal impact on C as L borrows to offset the share deficit, while an I-led expansion results causing  $v$  to rise. While the effects of  $v$  on  $\mu$  are relatively small, the secular increase in  $1-\mu$  and  $v$  increase  $f$ . This process repeats itself until increases in  $f$  overrun the gains in the gross profit share. At a critical juncture, net IK profit share is squeezed and I declines bringing  $v$  down.

Whether the economy cycles or crashes depends on bounding mechanisms in the model.

Here, small  $\partial C/\partial \mu$  implies that C plays a bigger role in sustaining the expansion ( $\frac{C}{Y}$  rises). As  $v$  declines from deficient I and  $f$  rises proportionately more than  $\mu$  declines, I declines and  $v$  declines further. As long as the rise in  $f$  and/or the decline in I is unbounded, the economy will decline precipitously.

Such a dynamic is exemplary of the super-predator/FK paradox. In more practical terms, unregulated FK activities result in undesirable outcomes in the form of an economy-wide crash.

#### d. Four Differential Equation Bounded Model

This likely outcome segues to a discussion of a bounded model. An FK PS induced crash is less likely if the upward tendency in  $f$  and significant declines in I and  $v$  are bounded. This is

respectively accomplished via 1) a quadratic restriction on  $\dot{f}$ , 2) the addition of a wealth effect influenced by pseudo FK-induced asset bubbles<sup>26</sup> and 3) stabilizing G policy.

With respect to 1) above, equation (2) is modified and renumbered (2').

$$\frac{\dot{f}}{f} = -\alpha_0 + \alpha_1(1 - \mu) + \alpha_2 v - \alpha_3(1 - \mu)^2 \quad (2')$$

where  $\alpha_3$  is a positive constant.

The squared term limits FK incursions on IK profits.<sup>27</sup> Thus, a continual rise in  $f$  underlying the FK paradox is limited and possibly reversed.

Wealth is treated as a second macro state variable.

$$\frac{\dot{W}}{W} = W_0 + W_1 f - W_2 f^2 + (1 - \beta_L) \mu \frac{Y}{W} - \beta_1 (\mu^* - \mu) \frac{Y}{W} \quad (7)$$

where  $W_0 - W_2$  are positive constants and the last two terms represent household saving and dissaving as changes in  $W$ .  $W_0$  includes the typical return on invested wealth.

The quadratic dependency of  $W$  on  $f$  limits the use of FK income for the purchase of secondary market assets and the price of those assets (at a given level of  $Y$ ). The  $f^2$  term accounts for an eventual tapering and decline in asset prices. Combined with the activation of the  $W$  effect in the  $C$  and  $I$  equations ( $\beta_2 > 0$  and  $Z_2 > 0$ ), asset bubble-induced increases in  $W$  reduce the decline in  $I$  and result in (unsustainable) increases in  $C$ .

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<sup>26</sup> Given that asset prices are not explicitly modelled, I use the term pseudo.

<sup>27</sup> An example would be limits on corporate stock buybacks.

Lastly, the use of stabilizing fiscal policy as in Equation (6) ( $G \neq 0$ ) places a possible upper and lower bound on  $v$ .<sup>28</sup>

The complete four DEQ bounded/extended model consists of equations (1, 2', 3 and 7) with  $\beta_2 > 0$ ,  $Z_2 > 0$ ,  $G$  as in equation (6),  $\beta_{IK} = 0$ , and  $Z_1 = 1$ . The dynamics of the model are similar to the basic model with both upper and lower bounding of the solution. Thus, instead of a likely crash, cyclical and chaotic outcomes are more likely.

#### IV. Qualitative Behavior: Unbounded Three Class Model

##### a. Underlying Dynamics

Unlike Goodwin's two class model that results in a stable limit cycle, the basic model is more likely to result in either equilibrium trajectories or unstable trajectories emanating from saddle points. This characteristic is desirable in explaining underlying secular trends.

In comparison to the Goodwin model, the major differences are an  $E=Y$  model<sup>29</sup> versus a growth framework for the macro state and the addition of a third class. In Goodwin,  $\frac{\dot{v}}{v}$  is determined by the profit rate ( $\pi_R$ ) minus the sum of productivity and labor supply growth rates. Thus, a decline in  $\pi_R$  from an increase in  $\mu$ , does not translate to an immediate decline in  $v$  until

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<sup>28</sup> Results below indicate that this type of policy is both bounding but also destabilizing. Typical state variable amplitudes increase. Thus, a dynamic control variable would be a better policy specification but would complicate the current model unnecessarily.

<sup>29</sup>An advantage of an  $E=Y$  model are: 1) the complexity of the macro model can be developed outside of the differential equation model which is more sensitive to complex specifications.

$\pi_R$  falls below the sum of the growth factors such that  $\frac{\partial v}{\partial \mu} < 0$ .<sup>30</sup> In contrast this derivative is strictly  $< 0$  here. With respect to Goodwin's  $\dot{\mu}$  equation, equation (1) is the same with the exception that it includes a  $(-\rho_2 f)$  term associated with a third class.

Consider a typical expansion in both models. Eventually as  $v$  rises enough,  $\mu$  increases but given the differences in  $\frac{\partial v}{\partial \mu}$ ,  $v$  continues to increase in Goodwin, but declines in our case unless offset by the behavior of a third class. In Goodwin,  $\mu$  rises enough over multiple periods to cause  $\pi_R$  and  $v$  to decline which ultimately results in a multi-period fall in  $\mu$  and a revival in  $v$ .

In our case, a small decline in  $v$  is enough to make  $\mu$  decline and  $v$  rise. A slight rise in  $v$  reverses the process and results in rapid saw tooth behavior in  $v$  and  $\mu$  that either continues or reaches equilibrium. It is only by the addition of a third class that qualitatively different behavior results. The possible qualitative outcomes are varied and conditioned by the relative strength of the three classes. These outcomes are considered below.

For example, consider relatively strong FK vis a vis IK and L. In the expansion, as  $\mu$  first declines and  $v$  rises, by equation (2'),  $f$  also declines till  $-\alpha_0$  is overcome. Thus,  $(1-\mu-f)$  increases and  $v$  rises as long as  $\frac{\partial Y}{\partial \mu} < 0$ . If  $f$  is slower to rise than  $\mu$  (depending on  $Y$  vs.  $\alpha_0$  and  $\rho_1$  vs.  $\alpha_2$ ),  $(1-\mu-f)$  and  $v$  continue to rise. Eventually,  $\mu$  and  $f$  both rise and  $(1-\mu-f)$  and  $v$  decline.<sup>31</sup> As  $f$  rises faster, eventually  $(1-\mu-f)$  decreases by enough that  $f$  declines. Here, the increase in  $1-\mu$  is

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<sup>30</sup> Goodwin's  $v$  dynamics could be replicated by subtracting the sum of growth factors from equation (3). Despite this, investment is still treated differently than in Goodwin as only having demand side impacts.

<sup>31</sup> Depending on whether  $f$  is falling slowly at this juncture,  $\mu$  can continue to rise at the same time that IK net profits and  $v$  fall via a reverse cost rationalization effect.

not enough to compensate for the rise in  $f$  and a sawtooth response is avoided. As a result,  $(1-\mu-f)$  continues to decline and  $v$  falls to a depressed equilibrium or crashes as a result of an FK PS crisis.<sup>32</sup> For later, it is important to note that in some circumstances  $f$  lags  $\mu$  dynamically.

#### b. Qualitative Behavior

Qualitative behavior is related to the relative power among the three classes. Table 2 summarizes seven alternative nexuses of class power that result in five different behaviors. References to the power of bi-lateral class relations as strong, moderate and weak are relative strengths.

Table 3 presents the parameter values associated with each case in Table 2. Case 1 is the base with parameter values reported in the first column. Subsequent columns contain only the values of parameters that have been altered with respect to the base case. Blank cells indicate that a parameter retains its base case value. Figure 2 depicts the state trajectories  $(\mu, v, f)$  and the time path for  $(1-u-f)$  for the five different qualitative behaviors. Parameterization of the model is derived from a rudimentary econometric analysis of behavioral relations reported in Appendix B.

Cases 1 and 2 exhibit our primary focus of this paper – an FK PS. In case 1 (2), the relationship between L vs. IK, FK vs. IK and FK vs. L are respectively weak, strong and strong (weak). These relations are consistent with a FK dominated profit-led regime. The dynamics of the system capture a FK PS crash. In case 1 (depicted in Figure 1), the economy crashes to  $v=0$ . This result is characteristic of the super-predator paradox.

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<sup>32</sup> This result is predicated on relatively weak L vs. IK where a small value for  $\rho_1$  ensures that  $f$  increases at a faster rate than  $\mu$  declines.

In Figure 1, case 1, it takes 19 periods for the economy to bottom. Using more realistic earmarks, it takes 10 periods for  $\mu$  to decline an amount equivalent to its drop between 1980-2015. This suggests that each time period is approximately 3.5 years. At  $t=10$ , states values are ( $v=.37, \mu=.52, f=.71, (1-u-f) = -.23$ ) and  $c=.85$  compared to the initial states/values respectively of  $(.9,.6,.042,.358)$  and  $c=.62$ . These trajectories follow the stylized trends highlighted in section II.<sup>33</sup>

G policy, as specified in equation (6), slows the decline but not the crash in the economy (not depicted). With policy,  $v$  reaches .001 at  $t=25$  instead of  $t=22$  in case 1. At  $t=10$ , the state vector is  $(.85, .52, .80, -.32)$  suggesting that in the short-run, policy slows the decline in the economy but is eventually overwhelmed. In addition, policy aids the ascent of  $f$ .

Table 2. Qualitative Behaviors in Unbounded Three Class Model

Case	L vs. IK	FK vs. IK	FK vs. L	Outcome
1	Weak	Strong	Strong	FK Profit Squeeze Crash
2	Weak	Strong	Weak	FK Profit Squeeze Crash
3	Strong	Moderate	Weak	RA Profit Squeeze – Stable Equilibrium
4	Strong	Moderate Strong	Weak	Double RA-FK Profit Squeeze to RA Profit Squeeze Stagnant Equilibrium
5	Strong	Moderate Strong	Strong	Double RA-FK Profit Squeeze to FK Profit Squeeze Crash
6	Moderate	Weak	Strong	Investment-Led Expansion to Stagnant Equilibrium with Under-C problem
7	Moderate	Weak	Weak	Wage-Led Expansion with Stable Equilibrium

<sup>33</sup> These outcomes are not very sensitive to borrowing by L to sustain C. For  $\beta_2 = 0$  and  $\beta_2 = 1$ , at  $t=10$  the two state vectors are respectively:  $(.35, .52, .70, -.22)$  and  $(.38, .52, .72, -.24)$  suggesting that L borrowing effects to maintain C are quantitatively small.



Table 3. Parameter Values for Unbounded Model Cases 1-7<sup>34</sup>

Parameter	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7
$\Upsilon$	0.004		0.0035	0.0035	0.0035		
$\rho_1$	0.00003		0.004	0.007	0.007	0.006	0.004
$\rho_2$	0.05		0.005	0.005	0.04		0.0008
$\alpha_0$	0.15			0.13	0.13		
$\alpha_1$	0.9		0.32	0.4	0.4	0.001	0.5
$\alpha_2$	0.1	0.000001	0.0001	0.001	0.01	0.0005	0.005
$Z_0$	700						
$Z_1$	1.0					0.4	
$\beta_0$	1500						
$\beta_1$	0.75						
$\beta_2$	0.75	0.0	0.75 and (0.0)	0.40 and (0.00 -1.00)		0.60 and (0.20)	0.0
$\beta_{FK}$	0.02						
$\mu^*$	0.6						
$Y_F$	10000						
$v^*$	(0.90)						
$A^e$	(0.40)						
Initial conditions							
$\mu(0)$	0.6						
$f(0)$	0.042						
$v(0)$	0.9						

<sup>34</sup> Values in parentheses are not used in the main simulation. They are used in additional sub-cases discussed in the text.

Case 2 with FK weakened against L and no policy exhibits qualitatively similar behavior to case 1. In this scenario, the decline is slowed and the ultimate crash ( $v=.001$ ) occurs at  $t=25$ . At  $t=10$ , the state vector is (.61, .54, .33, .12). Thus, the decrease in FK power softens the early decline in  $v$ , marginally slows the decent in  $\mu$ , more substantially deters the rise of  $f$  and thus the decline in IK net profits.

Turning to a stable RA profit squeeze, reported as case 3, L is significantly strengthened versus IK and FK and FK remains moderately strong with respect to IK, a RA profit squeeze occurs.<sup>35</sup>

The strength of L vs. IK is expressed by  $\gamma$  and  $\rho_1$ . Here,  $f \rightarrow 0$ ,  $\dot{\mu} > 0$  until  $v$  equilibrium ( $v_e$ ) ( $v=.875$ ). While an unemployment rate of 12.5 percent may seem excessive prior to a fall in  $\mu$ , the relevant conceptual rate should be a broad measure of unemployment such as U6 which is often double the official rate.<sup>36</sup>

Case 3 considers a stable secular RA profit squeeze with an acceptable equilibrium that includes an endogenous decline in  $f$ . As  $f \rightarrow 0$ , an effective floor is placed under IK net profits allowing the economy to prosper. The equilibrium state vector is ( $v=.875$ ,  $u=.70$ ,  $f=0$ ,  $1-\mu-f=.30$ ). Determining the time scale as above,  $t = .21/.25$  years. Thus, the expansion in  $v$  is approximately 5.75 years. During the first 200 periods, while  $f > 0$  but declining, the increase in  $\mu$  is aided by a reversal of the  $\rho_2 f$  effect.

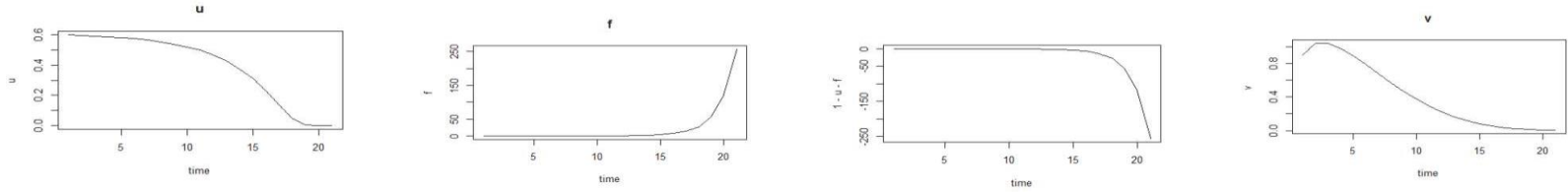
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<sup>35</sup> Given that  $\mu > \mu^*$  in equilibrium, results are simulated for both  $\beta_2=.75$  and  $\beta_2=0$ . The former results are reported. In that case, debt retirement reduces  $C$  and mildly bounds the outcomes, but the results are qualitatively the same in both cases. For  $\beta_2 = 0$ , the equilibrium state vector is ( $v=1$ ,  $\mu=.887$ ,  $f=0$ ,  $1-\mu-f=.123$ ) and  $v$  peaks at 1.28. after a 15.8 - 18.0 year expansion.

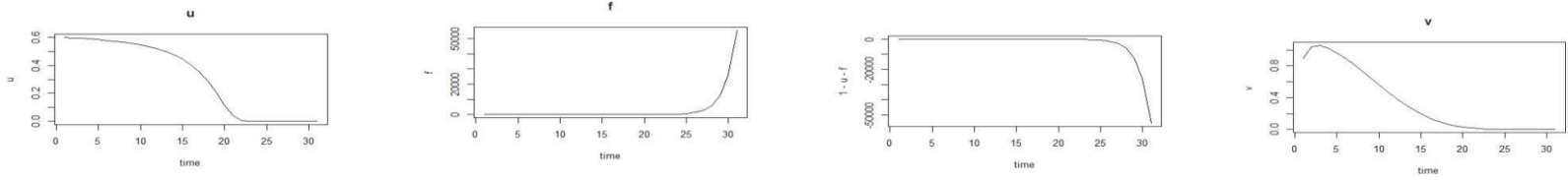
<sup>36</sup> U6 is used in the econometric estimation of parameters along with U3.

Figure 2. Unbounded Model Cases

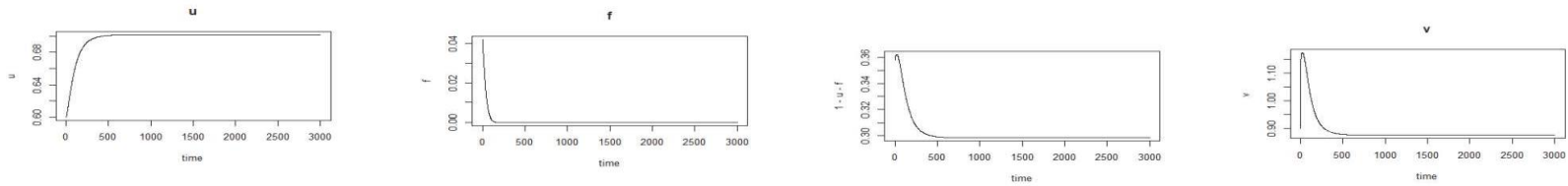
CASE 1. FK PS



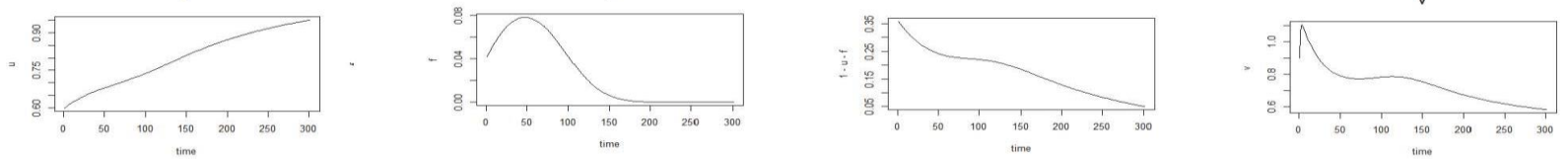
CASE 2. FK PS WEAKENED FK



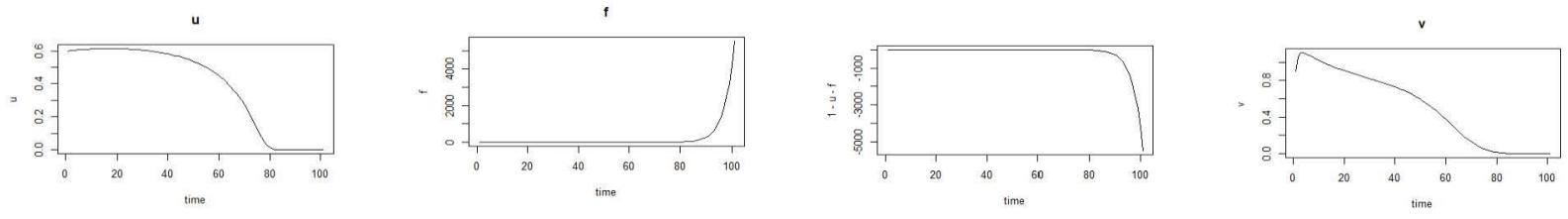
CASE 3. RA PS



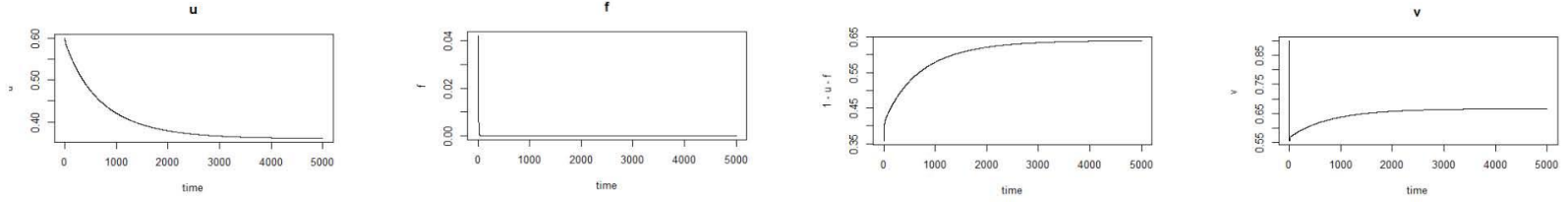
CASE 4. FK-RA PS WITH TRANSITION TO RA PS



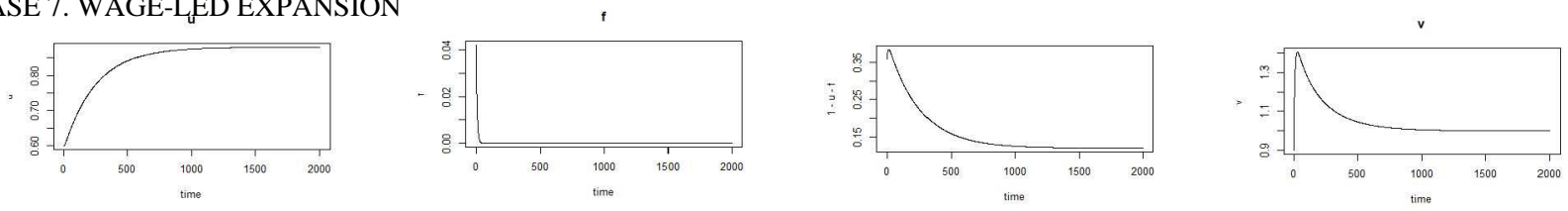
CASE 5. FK-RA PS WITH TRANSITION TO FK PS



CASE 6. I-LED EXPANSION WITH UNDER-CONSUMPTION



CASE 7. WAGE-LED EXPANSION



In this case, where FK ultimately has no impact on the distribution of income, a more stable and acceptable outcome results despite having strong L in a profit-led regime.

Case 4 depicts a double FK-RA PS that transitions to a RA PS produced depressed equilibrium. In this case, power relations are L vs. IK strong, FK vs. IK moderately strong and FK vs. L weak. These relations provide FK with the ability to aid in the PS but ultimately let L assume the dominant role.

Given that  $\mu > \mu^*$  throughout, the appropriate value of  $\beta_2$  must be considered. On one hand,  $\beta_2 = 0$  (debt reduction does not reduce C) allows for a more robust economy. On the other hand,  $\beta_2 > 0$  can be used to mildly bound state variables. Additionally, for  $\beta_2 \leq .4$ , a second cycle in  $v$  occurs after a short transient cycle. As a result, we set  $\beta_2 = .4$ .<sup>37</sup> The relevance of deleveraging behavior also suggests that  $\beta_2 > 0$  is realistic.<sup>38</sup>

Using the same method for determining time increments, one period is approximately .5 years. The resulting double FK-RA PS lasts 25 years until  $f$  starts to decline.<sup>39</sup> After that point  $\mu$  continues to rise faster than  $f$  declines further reducing IK net profits which leads to further declines in  $v$ . The end result is a depressed equilibrium with a state vector of ( $v=.502$ ,  $\mu=1.04$ ,  $f=0$ ,  $1-u-f=-.04$ ). The expansion is not sustainable once  $(1-\mu-f)$  declines more rapidly. Compared to case 3, the increased power of FK further destabilizes the economy.<sup>40</sup>

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<sup>37</sup> All values of  $\beta_2$  in the range 0-1 produce qualitatively similar results except for the second cycle.

<sup>38</sup> In order to clearly present details of interest, the associated graphs, for this case, do not depict the final state equilibriums.

<sup>39</sup> Despite the weak revival in  $v$  at that time, the steep decline in IK gross profits forces  $f$  to decline.

<sup>40</sup> The second (weak) expansion in  $v$  occurs after 37 years and is the result of faster increases in  $\mu$  than declines in  $(1-\mu-f)$  which slow around that time.

Starting from the previous nexus of class relations, if FK vs. L is strengthened (via increases in  $\rho_2$  and  $\alpha_2$ ) a double profit squeeze that degenerates into a FK PS crash results as reported in case 5.<sup>41</sup> The time increment is one year.  $\mu$  increases for 17 years although minimally from .600 to .613, while  $f$  increases continuously at increasing rates. Once  $\mu$  declines, the steeper ascent of  $f$  dominates and leads to a steeper decline in IK net profit. Eventually, an FK PS crash takes over with  $v=u=0$  and  $f$  and  $1-u-f$  exploding respectively upward and downward.

An additional weakening of L, this time vis a vis IK, a strengthening of IK vs. FK and a weakening of L vs. FK, leads to an I-led expansion after a brief transient decline in  $v$  (case 6). In order to avoid an explosive outcome, debt financed  $C$  is mildly reduced ( $\beta_2 = .6$ ) to enable under-consumption tendencies to evolve and IK saving out of net profits is introduced ( $Z_1 = .4$ ). The expansion ends in an undesirable equilibrium with  $v=.68$ ,  $\mu=.365$ ,  $f=0$ , and  $1-\mu-f=.635$ . After a transient increase in  $c$  to .74, it declines to .645. Thus, under-consumption tendencies push the economy to undesirable levels. Further declines in borrowing ( $\beta_2 = .2$ ) lead to a lower level of performance ( $v=.45$ ,  $\mu=0$ ,  $f=0$ ,  $1-\mu-f=1$ ) as a result of more serious under-consumption problems.

Finally, case 7 predicated on moderate L vs. IK and weak FK vis a vis IK and L results in a wage-led<sup>42</sup> stable expansion at acceptable state variable levels. The equilibrium vector is ( $v=1$ ,  $\mu=.88$ ,  $f=0$ ,  $(1-\mu-f)=.12$ ).

In sum, more moderate class power relation and a neutralization of FK's role on the distribution of income produce more stable and desirable outcomes. Increased FK power and/or

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<sup>41</sup> Given that the rise in  $\mu$  is limited before declining,  $\beta_2$  is left at its base value of .75.

<sup>42</sup> Technically this is not a wage-led economy because  $\partial Y/\partial \mu < 0$ , but  $C$  plays a more prominent role.

an imbalance in class relations with strong L and strong FK (double PS) result in more instability and less desirable outcomes.

## V. Qualitative Behavior Bounded Three Class Model: From Crashes and Depressed Equilibrium to Chaos and Cycles

### a. Modified Underlying Dynamics

The bounded model adds wealth/bubble dynamics along with restrictions on W behavior, a wealth effect on expenditures, limitations on f growth and countercyclical government policy. The most important additions are the limitations placed on both f and W which take the form of diminishing marginal growth with respect to their main drivers (respectively  $(1-u)$  and f).

Returning to the above discussion of a typical expansion, starting in the early-mid expansion when  $\mu$  increases, f is still declining, W increases exogenously and  $(1-\mu-f)$  increases. Eventually f rises slowly causing W to increase further and  $(1-\mu-f)$  to decline. Here, the decline in I is offset by a wealth-induced rise and a  $\mu$ -induced rise in C and v continues to rise. Without the f and W interaction, there would be a short cycle where v would decline as a result of increases in  $\mu$ . Alternatively, the increase in v allows both f and W to rise further, but at a slow rate due to limitations and large  $\alpha_0$  values. As long as the dual C effect offsets the decrease in I, v rises but if I dominates, v declines causing  $\mu$  to decline. If  $\mu$  falls faster than f increases,  $(1-\mu-f)$  may increase for a while, thus continuing the slow expansion. Finally, f rises fast enough so that  $(1-\mu-f)$  declines and W is restricted enough such that investment declines dominate and a slow contraction joins with the fast contraction.

## b. Technical Aspects of the Bounded Model

Three attributes of the four DEQ model in equations (1,2',3 and 7) contribute to the qualitative behavior of the system. The model can be characterized as: 1) a coupled set of two nonlinear oscillators with endogenously determined coupling strength; 2) an overall cyclical environment consisting of two component cycles of which one is a fast cycle and the other a slow cycle; and 3) two equations of motion (2' and 7) that are quadratic in state variables allowing for the possibility of an equilibrium state vector that is not contained in real space ( $R^4$ ). These characteristics underlie state solutions ranging from damped cycles to limit cycles to increasing amplitude limit cycles to bursting and chaotic trajectories.

Examples of coupled oscillators include the synchronized chirping /bio luminescence of crickets/fire flies.<sup>43</sup> In our case  $\dot{v}$  and  $\dot{\mu}$  comprise one nonlinear oscillator, while the interaction between  $\dot{f}$  and  $\dot{W}$  the other. The former comprises the RA PS dynamic while the latter implicitly captures the cycle in asset prices (wealth).<sup>44</sup> Abstracting from  $\mu$  and  $v$  movements, exogenous increases in  $f$  in the case where  $-\alpha_0 > 0$  results in increased trading on secondary markets by FK causing asset prices to rise. At some point, asset prices rise slowly and eventually decline causing a cycle in  $W$ .

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<sup>43</sup> A classic physics example is two identical pendulums attached by a spring. Starting from rest, if both pendulums are moved simultaneously to the left or right without altering spring tension, then the two cyclical trajectories will have synchronized peaks and synchronized troughs. Alternatively, if the two pendulums are spread apart and released, the peak (trough) of one cycle is aligned with the trough (peak) of the other. Other joint behavior is also possible.

<sup>44</sup> The FK PS only emerges once both subsystems interact.



The two oscillators are coupled via their joint dependency.  $f$  directly affects  $\mu$  and  $v$  and  $W$  directly impacts  $v$ . In return,  $\mu$  and  $v$  affect both  $f$  and  $W$ . Critical coupling parameters associated with unit changes in relevant state variables are:  $\alpha_2$ ,  $\rho_2$ ,  $(1-\beta_L)\mu Y_F$ ,  $\beta_1 v Y_F$ ,  $-\beta_1(\mu^* - \mu)Y_F$ ,  $2\alpha_3(1-\mu) - \alpha_1$  and the partial derivatives of  $A$  with respect to  $\mu$  and  $f$ . It should be noted that numerous coupling parameters are endogenous. In particular, when  $\mu$  and  $v$  increase (decrease) relevant coupling parameters increase (decrease) with the exception of the  $\mu$  effects on  $\dot{f}$  which depend on the relative size of  $\alpha_3$ ,  $\alpha_1$  and  $(1-\mu)$ . At extreme values of  $\mu$ , it is possible for this component of coupling strength to decrease, but for most realistic values coupling strength increases.

Considering the coupled oscillators as a whole, particularly with respect to key macro state variables  $v$  and  $W$ , an interesting aggregate cyclical environment exists. At the same time that  $\dot{f}$  and  $\dot{W}$  equations produce (unsustainable) counteracting tendencies to FK PS and FK-RA PS crashes, their interaction is responsible for adding a sub harmonic cycle within the overall unified cycle. In other words, cyclical solutions are actually made up of a fast, fundamental cycle primarily based on  $\mu$  and  $v$  (RA PS) and a sub-harmonic slow cycle related to  $f$ ,  $W$ , and  $v$  interactions (FK PS). The quadratic terms in  $\dot{f}$  and  $\dot{W}$  help establish not only bounded state trajectories but also a slow cycle.<sup>45</sup> The determination of  $\mu$  in the fast cycle feeds back to the slow cycle by impacting  $(1-\mu)$  which influences  $f$  and  $W$ . Despite mathematicians' use of fundamental and subservient descriptors for the two cycles, the secular weakening of  $L$  underlies

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<sup>45</sup> The  $-\alpha_0$  term in equation (2') further reduces the pace of the cycle.

its fast nature and subservient economic role in overall behavior and the FK PS component becomes the dominant cycle.

Lastly, the inclusion of quadratic state variables in some DEQs implies that the equilibrium state vector may include complex conjugate numbers. In this case, an equilibrium does not exist in real space ( $\mathbb{R}^4$ ). Consideration of such equilibria has only recently become a topic of interest referred to as hidden attractors (Leonov and Kuznetsov(2013)).

These three characteristics of the bounded model have important implications for the qualitative behavior of the system. The coupling of two nonlinear oscillators creates the potential for either two interacting limit cycles or chaotic trajectories. In general, findings on increased coupling strength suggest that the two cycles/trajectories are more likely to become synchronized.<sup>46</sup> In our context, endogenous increases in coupling strength as  $v$  and  $\mu$  increase will synchronize cycles in  $\mu$ ,  $v$ , and  $f$ . Simultaneous increases in  $\mu$  and  $f$  lead to a potent double profit squeeze with larger amplitudes and a more dramatic reversal. This makes a transition from an overall limit cycle to limit cycles with larger amplitudes and potential turbulent behavior once the bounds on state variables are hit more likely.

The aggregated cycle consisting of both a fast and slow cycle, also referred to as a dual time scale problem, is also a source of potential turbulence. Berge' et al. (1984; Ch. 9) shows that the combination of such cycles in general has a tendency to form limit cycle – like behavior subject to turbulent/chaotic bursts. As a precursor of chaos, Berge' et al. (1984; p. 249) argue

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<sup>46</sup> This result is somewhat intuitive. If two qualitatively similar trajectories are only mildly coupled, the potential for differences in one trajectory to dominate the interaction and alter the overall behavior is more likely. Surprisingly, this result has been found to hold for two chaotic trajectories (Boccaletti et al. (2002)).

that the amplitudes of the fundamental (fast) cycle decline while those of the subharmonic (slow) cycle increase. Starting from a limit cycle as a base solution, if the latter dominates (as in our case), the same increased amplitude behavior predicted by increased coupling strength emerges. Berge' et al. view a turbulent response in this situation as the result of competition between the driving and restorative system forces that results in unexpected outcomes. This particularly pertains to our model as the restorative forces have abrupt impacts once they are strong enough to lower  $v$ .

Finally, the recent analysis of non-equilibrium (hidden) attractors (Leonov and Kuznetsov (2013)) offers another explanation for potential chaotic results. Given the newness of this research and the lack of general results, we do not pursue this line of research.

These properties of the model suggest that the prospective overall qualitative behavior of the state variables is likely to experience transitions, typically as a result of bifurcations, from damped cycles to limit cycles as a base to modified limit cycles characterized by increasing amplitudes to bursting and chaotic behavior. Focusing more on the disaggregated behavior of components, we would expect to find two separate limit cycles. One in  $W, f$  state space and one in  $\mu, v$  state space that combine to form a single aggregate limit cycle with increasing amplitudes. This result is best viewed in two dimensional phase planes that cross between both coupled systems: in  $(\mu, f)$ ,  $(v, f)$ ,  $(\mu, W)$  and  $(v, W)$  spaces. We also expect, due to endogenous coupling strength and fast and slow cycles to observe limit cycles with ever increasing amplitudes at higher values of  $\mu$  and  $f$ . Also, a quick reversal in behavior back to smaller amplitude limit cycles is likely once the bounds for the system are encountered. This quick reversal in the system can act as a source for bursting and chaotic behavior. Finally, a far-away view of state

trajectories should reveal both the fast and slow cycles that underlie the overall cyclical behavior as demonstrated in Berge' et al. (1984; p. 249).

### c. Qualitative Behavior

In this section, two qualitative behaviors from the unbounded model that are most relevant to the NL era – FK PS and FK-RA double PS – are analyzed. For each of the two PS, the cases presented consider a continuum of qualitative behaviors. As just discussed, the qualitative behavior associated with each PS lies on a continuum from damped cycles to limit cycles, comprised of a fast and slow cycle, to limit cycles with increasing amplitudes and to irregular limit cycles that experience chaotic bursts.

All simulations analyzed are reported in Table 4 based on the relative bi-lateral strengths of the three classes, other model characteristics and key results. Parameter values for each case are reported in Table 5. Some of the more technical results discussed in the preceding section are reported in Appendix C.

Parameters in the bounded model are not directly comparable and thus are likely to deviate from parameters in the unbounded model. Finally, quadratic parameters take on larger-than-life values in order to incorporate a continuous downturn in asset prices/wealth that typically would take the form of a discontinuous free-fall when a bubble bursts.

## FK PS Results

For the FK PS case, there are two distinct paths leading to the described continuum of outcomes. There is a bubble chaotic path and a weakened FK path. In the latter case, as FK is moderately weakened, via decreases in either  $\alpha_1$  or  $\alpha_2$  or an increase in  $\alpha_3$ , the FK PS perhaps unexpectedly becomes a FK-RA PS which exhibits chaos. This result is associated with a large increase in  $\mu$  and a moderate increase in  $f$ .  $\mu$  increases more due to the weakening of FK. Now, smaller changes in  $\frac{\dot{f}}{f}$  with respect to  $v$  occur and relatively  $\frac{\dot{\mu}}{\mu}$  via a slow growing  $\rho_2 f$  term in equation (1). Both states increase on average as larger  $v$  amplitudes more heavily weighted on the upside result in increases in  $v$  driving both  $\mu$  and  $f$  up. Finally,  $\mu$  and  $f$  cycles become synchronized in a way that further underlies the double PS. In particular, the  $\mu$  peak aligns with the trough in  $f$ . Thus, as  $\mu$  declines,  $f$  rises such that  $f$  increases when  $\mu$  is still close to its peak resulting in the double PS. Also,  $v$ ,  $\mu$  and  $W$  peaks are aligned with the trough in  $(1-\mu-f)$  allowing for expenditures and income to remain in near balance even at I and C extremes. This result is addressed in Case 5.

The bubble path to chaos results from increases (decreases) in  $W_0$  ( $W_1$ ). This corresponds to less bubble prevention mechanisms/institutions. In this scenario, the center point for  $\mu$  and  $v$  around its fluctuations remain the same,  $v$  remains the same but  $v$  experiences larger amplitudes. The timing of  $v$  fluctuates and allows for a moderate overall increase in  $f$  around its fluctuations and a moderate decline in  $(1-\mu-f)$ . At the same time  $W$  increases substantially by

Table 4. Qualitative Behaviors in Bounded Three Class Model

Case	L vs. IK	FK vs. IK	FK vs. L	Absolute FK	Outcome
1	Weak	Strong	Moderate Strong		FK PS Damped cycles. Cycles die out after 50 years. Acceptable (unsustainable) equilibrium. $f$ is large and $\mu$ is low.
2	Weak	Strong	Moderate Strong	Increase	FK PS limit cycle with $v$ center pt. the same as $v_e$ above. $f \uparrow$ , $W \uparrow$ . <u>Volatility increases.</u>
3	Weak	Strong	Moderate Strong	Increase More	Limit cycle base with irregular extreme values departures in $v$ and $W$ . Extreme values cycle above limit cycle extreme. $W \uparrow$ , $f \uparrow$ . Volatility increases.
4	Weak	Strong	Moderate Strong	Increase Further	Limit cycle base with upper bursts in $W$ and $v$ trajectories. $f \uparrow$ , $W \uparrow$ , $v \uparrow$
5	Strong	Moderate	Strong		Double FK-RA PS with erratic cycles. $W \downarrow$ , $\mu \uparrow$ , $f \uparrow$ marginally.
6	Moderate	Moderate Strong	Strong		FK-RA PS chaotic behavior with significant volatility. Undesirable $v$ level and distribution of income.
7	Moderate	Weak	Weak		FK-RA PS damped cycles. Cycles die out at $t=1000$ . Other characteristics the same as case 6.

Table 5. Parameter Values for FKPS and FK-RA PS Cases

FKPS CASE	$\gamma$	$\rho_1$	$\rho_2$	$\alpha_0$	$\alpha_1$	$\alpha_2$	$\alpha_3$	$Z_0$	$Z_1$	$Z_2$	$\beta_0$	$\beta_L$	$\beta_I$	$\beta_2$	$\beta_{KK}$	$\mu^*$	$W_0$	$W_1$	$W_2$	$Y_T$	$v(0)$	$\mu(0)$	$f(0)$	$W(0)$
1	.025	.025	.005	.10	1.7	.25	4.3	-100	1.0	.02	1000	.75	.75	.05	.02	.60	.10	.60	5.0	10,000	.90	.60	.042	27,000
2																		1.8						
3																		2.95						
4																		3.8						
5				.10																				
FK-RA PS CASE																								
1	.007	.04	.008	.10	.60	.5	4.0	500	1.0	.02	-800	.75	.75	.05	.02	.60	.05	.35	.47	10,000	.90	.60	.06	27,000
2													.47											

doubling at its center point. Here, the larger wealth effect can propel trajectories beyond their limit cycle extremum. The continuum of bubble path behavior is presented in Cases 2-4.

FK PS case 1, is summarized in Table 4, depicted in Figure 3 with parameter values listed in Table 5. Bi-lateral class relations associated with a FK dominated profit-led regime and relevant for this case are: weak L vs. IK, strong FK vs. IK and moderately strong FK vs. L. The

stabilizing effects incorporated in the bounded model result in damped cycles with an equilibrium vector of  $(v=1.040, \mu=.526, f=.199, W=27.4, (1-\mu-f)=.275)$ . Given the equilibrium value of  $(1-\mu-f)$ , the PS is mild. The cycles around  $v_e$  suggest that further upward bounding is desirable. The time interval is calculated based on the 76 periods it takes for  $\mu$  to go from .62 to a value of .52 +/- .001. Using a 35-year period for such a decline in  $\mu$ , each time period is approximately .5 year. The state variables at this juncture are  $(v=.997, \mu=.522, f=.186, (1-\mu-f)=.291)$ . The length of the first two cycles in  $v$  from peak to peak are approximately 15 years.

The trajectories capture the secular trends associated with a FK-dominated profit-led regime:  $\mu$  declines,  $f$  rises,  $v$  remains close to full employment after a major crisis,  $(1-\mu-f)$  declines and  $c$  increases. While the equilibrium for  $v$  is desirable, it is predicated on an undesirable redistribution of income from L and IK to FK. Yet the rise to power of FK is likely to propel the economy down a path to more unsettled and unsustainable behavior. If FK power is capable of reducing bubble prevention policy/regulation, a limit cycle or chaos can result (see cases 2 and 3 below).

In addition, attempts to redistribute income within an investment-led framework requires careful consideration. Increases in  $\mu$  and decreases in  $f$  must be balanced so as to produce desirable levels of IK net profits to maintain  $I$  and  $v$ . Alternatively, a transition to a wage-led regime requires less constraints on redistribution.

When an asymmetric G (anti-inflation) policy<sup>47</sup> is implemented to stabilize  $v$  at its upper bounds (not reported or depicted), equilibrium values remain the same, but a significant number

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<sup>47</sup> This is the same G policy expressed in equation (6) except that it is only implemented when  $v > v^*$ .

of additional cycles prior to equilibrium occur. Thus, this policy is mildly destabilizing suggesting that a dynamic policy control variable would be more appropriate.

Other qualitative FK PS behaviors for the bounded model are associated with the two paths to limit cycles and chaotic outcomes. For the bubble path, increases (decreases) in  $W_1$  ( $W_2$ ) lead to this result. As  $W_1$  increases from .6, the number of cycles prior to equilibrium increase. At  $W_1 = 1.8$  a limit cycle first appears for all state variables. These trajectories are depicted in Figure 3, case 2. The absolute rise in FK power increases volatility in the economy, while increasing  $f$  resulting in the following approximate center points for state variables ( $v=1.040$ ,  $\mu=.53$ ,  $f=.27$ ,  $(1-\mu-f) = .10$ ). Even at the initiation of limit cycles, slight irregularities can be seen at the upper boundaries of the  $W$  cycle. These figures are extremely useful for highlighting the underlying fast cycles (in dark) and slow cycles (in gray) that underlie the limit cycle. As discussed in section V.b, the declining amplitude of the fast cycle at the same time that slow cycle amplitudes increase is indicative of an impending transition to chaotic behavior.

As  $W_1$  increases further, the irregularities in  $W$  become more pronounced and begin to appear in the upper reaches of other trajectories. At  $W_1 = 2.95$ , the tops of the  $v$  and  $W$  trajectories show cycles above limit cycle extremum (depicted in Fig 3., case 3). At  $W_1 = 3.8$ , the trajectories take on a spikey look associated with chaotic bursts (Fig 3. case 4).

The other path to chaos via a weakening of FK vs. IK (smaller  $\alpha_1$  or  $\alpha_2$ ) results in a double PS. Starting from FK PS case 1 above, between  $1.15 \leq \alpha_1 \leq 1.7$ , the number of cycles till equilibrium increases. For  $.8 \leq \alpha_1 \leq 1.1$ , limit cycles begin in all state trajectories with the  $W$  path exhibiting minor irregularities at its maximum reaches. For  $\alpha_1 \leq .8$ , all state trajectories



experience limit cycles with irregularities. For  $\alpha_1 = .1$ , an irregular pattern is clear in all states as depicted for  $v$  and  $W$  in figure 3., case 5. Here, a double PS results. The weakening of FK indirectly strengthens L, while FK maintains moderate strength vs IK. As a result, profits are squeezed from two directions.

#### FK-RA PS Results

For the case of strong L vs. IK, moderately strong FK vs. IK and strong FK vs. L, an underlying limit cycle base made up of fast (RA PS) and slow (FK PS) cycles experiences the described continuum of behavior. In all cases, a double PS takes place. The parameter values that serve as the base case reported in Table 5 as case 6, clearly indicate that the FK portion of the double squeeze dominates. Not only are the  $f$  amplitudes more substantial, but the impact of changes in relevant state variables ( $\mu$  and  $v$ ) on  $f$  exceed those impacts for  $\mu$ .

Case 6 considers the chaotic result which is depicted in Figure 3. Average values of state variables around their cyclical fluctuations are approximately ( $v=.3$ ,  $\mu=.8$ ,  $f=.65$ ,  $W=55$ ,  $(1-\mu-f) = -.60$ ). The relative strength of L and FK vis a vis IK leads to large income shares for the two classes and a negative IK share that brings an I-led economy to fluctuate around depressed levels.

The secular trends are for  $\mu$ ,  $f$ ,  $W$  and  $c$  to increase and  $(1-u-f)$  and  $i$  to decline. The economy is maintained at inadequate employment levels by rising  $c$  as a result of increased L income and increased wealth-induced C. This outcome is more sustainable than the FK PS outcome because L debt is not necessary, but the outcome is still dependent on unsustainable pseudo asset bubbles.

A key distinction exists in this double PS case with respect to the synchronization of  $f$  and  $\mu$ . In the FK PS the troughs and peaks of  $f$  and  $\mu$  were aligned. For the FK-RA PS in case 6, the peaks of both states are virtually synchronized.<sup>48</sup> This is predicted by the factors that influence coupling strength discussed in section V.b.<sup>49</sup> Thus,  $\mu$  and  $f$  rise in unison causing the double PS.

An average level  $\mu > \mu^*$  requires consideration of  $\beta_2$  values. For  $\beta_2 > 0$ , a drag on  $C$  from debt retirement (deleveraging) operates. In case 1,  $\beta_2 = .75$ . Considering the significant role that consumer deleveraging played in the post 2008 period, this case may be realistic. If  $\beta_2 \leq .45$ , the qualitative nature of the solution changes to damped cycles.<sup>50</sup> Case 7 shows results for  $\beta_2 = .3$ . Here, additional  $C$  has a stabilizing effect on volatility but not on the level of the economy. Equilibrium values are similar to the average values above ( $v=.33$ ,  $\mu=.77$ ,  $f=.785$ ,  $W=52$ ,  $(1-\mu-f)=-.56$ ). Equilibrium is not reached until after 1000 periods. Thus, realistically a cyclical solution exists. Stabilization of the economy at a desirable  $v$  equilibrium requires a careful redistribution of power. A significant decrease in FK power and a moderate decline in L power are needed. In an I-led mode, the redistribution must be managed wisely. In order to prevent unrealistic increases in  $v$ ,  $\mu$  and  $f$  must be used to temper I activity to bound  $v$ .

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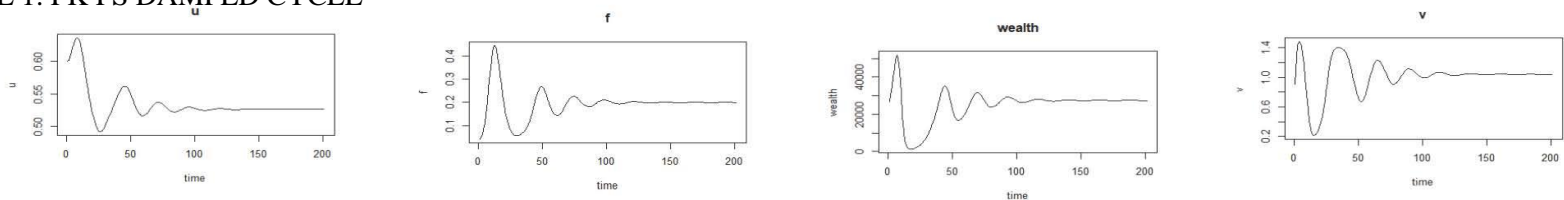
<sup>48</sup> The occurrence of  $\mu$  and  $f$  peaks for seven arbitrarily chosen time periods are respectively  $t=21,23$ ;  $t=77,80$ ;  $t=123,126$ ;  $t=203,207$ ;  $t=509,511$ ;  $t=594,597$ ; and  $t=903,905$ .

<sup>49</sup> The higher values, but not extreme values, of  $\mu$  in the latter case likely increase coupling strength more than lower values of  $v$  decrease it. Thus, more synchronization is expected.

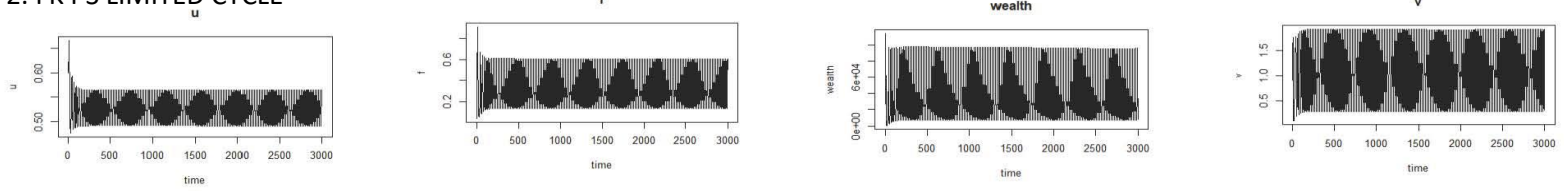
<sup>50</sup> Between  $.3 < \beta_2 < .75$  the continuum of behavior described above is experienced.

Figure 3. Bounded Model Cases

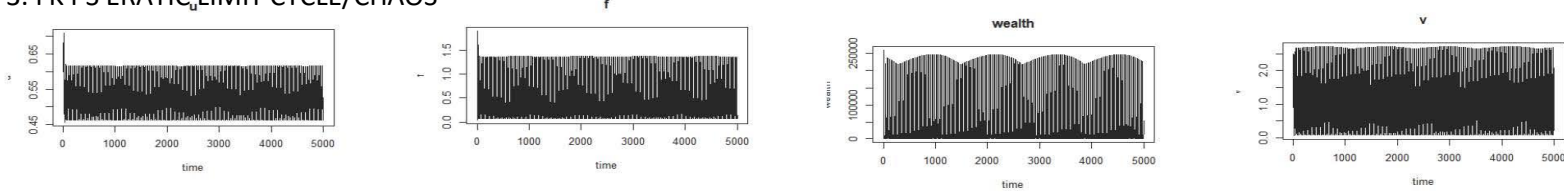
CASE 1. FK PS DAMPED CYCLE



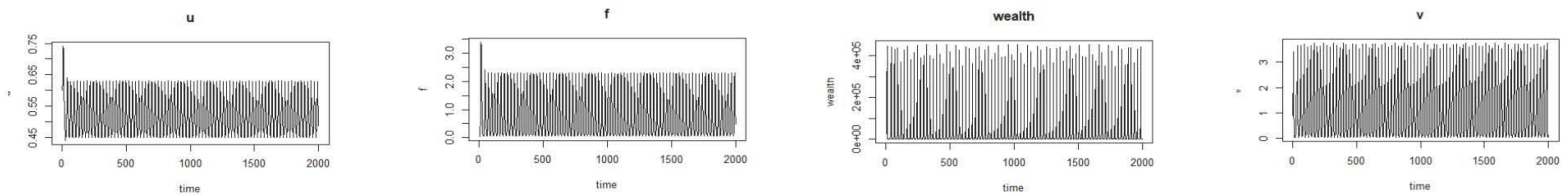
CASE 2. FK PS LIMITED CYCLE



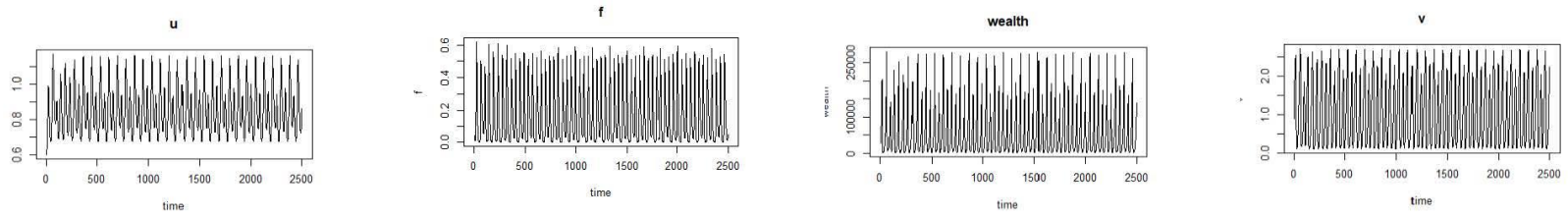
CASE 3. FK PS ERATIC LIMIT CYCLE/CHAOS



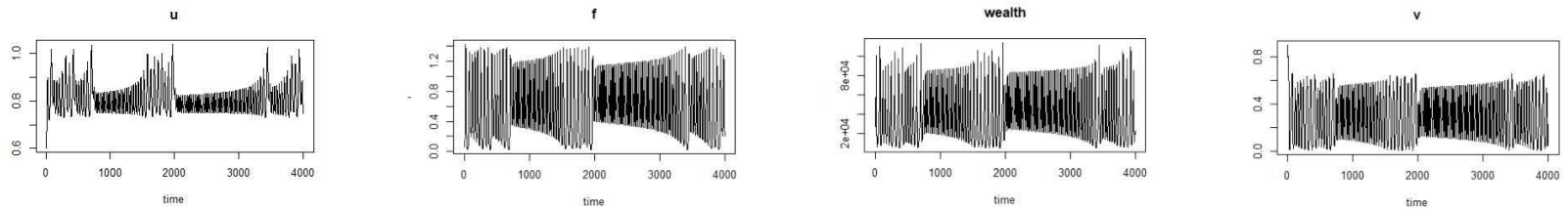
CASE 4. FK PS ERATIC LIMITED CYCLE/CHAOS



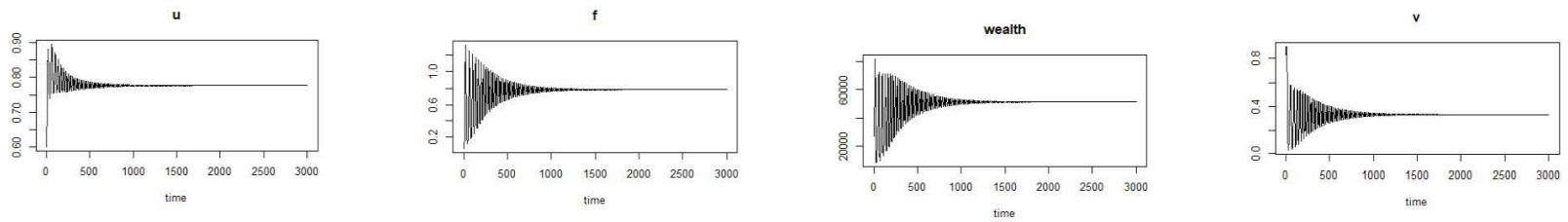
CASE 5. WEAKENED FK TRANSITION TO DOUBLE PS



CASE 6. FK-RA PS CHAOS



CASE 7. FK-RA PS DAMPED CYCLE



The addition of a symmetric G policy (with  $\beta_2 = .75$ ), also produces damped cycles with the same unacceptable equilibrium vector. The number of cycles until equilibrium increases in this case.  $\beta_2 = 0$  does not alter this solution (not reported).

## VI. Sensitivity Analysis

Results of a sensitivity analysis are reported in Appendix D, Tables D.1 and D.2 respectively for the FK PS and FK-RA PS solutions and are summarized here. Sensitivity analysis determines the robustness of chaotic behavior discussed above and it highlights which parameters are related to the continuum from damped cycles to limit cycles to limit cycle with irregularities to chaos.

For the FK PS case, the continuum from damped cycles (base case) to more volatile outcomes are related to increases in  $W_0, W_1, \alpha_3$  and  $\beta_L$  and decreases in  $\alpha_1, \alpha_2$  and  $W_2$ , while for the most part  $Z_1, Z_2$  and  $\beta_{FK}$  values do not alter the damped cycle result. The impact of  $W_0$  and  $\beta_L$  influence volatility resulting from excess demand,  $W_1$  and  $W_2$  increase the absolute power of FK and thus the intensity of the PS and  $\alpha_1 - \alpha_3$  weaken FK pushing the solution to a double PS. In the latter case, FK power is only moderately reduced while at the same time L is strengthened vs. FK. One key insight is that unbalanced power relations are responsible for the lack of stability in the economy.

Altering one parameter at a time, chaotic results are shown to exist for the following parameter ranges:  $.3 \leq W_0 \leq .89$ ,  $W_1 > .22$ ,  $.17 < W_2 \leq .6$ ,  $.32 \leq \beta_L \leq .99$ ,  $0 \leq \alpha_1 \leq 1.15$ , and  $5.3 \leq \alpha_3 \leq 7.85$ .

These results are robust over a wide parameter range related to the FK PS and double PS.

For the double PS, the path to less stable solutions is related to increases in  $\rho_1$ ,  $\alpha_1$ ,  $\alpha_2$ ,  $W_0$ ,  $W_1$ ,  $W_2$ ,  $Z_2$ ,  $\beta_L$ ,  $\beta_{FK}$  and decreases in  $\gamma$ ,  $\rho_2$ ,  $\alpha_0$ ,  $\alpha_3$  and  $\beta_1$ . The impact of  $\gamma$ ,  $\rho_1$  and  $\rho_2$  strengthen labor's ability to squeeze profits, the  $\alpha_0 - \alpha_3$  and  $W_0 - W_2$  effects strengthen FK either absolutely or vis a vis IK and thus the FK PS component. The  $\beta_L$ ,  $\beta_{FK}$ ,  $\beta_1$  and  $Z_2$  effects increase demand volatility. Chaotic parameter ranges are much more narrow in this case:  $.0062 \leq \gamma \leq .0073$ ,  $.039 < \rho_1 \leq .05$ ,  $.007 \leq \rho_2 \leq .009$ ,  $.09 \leq \alpha_0 \leq .1$ ,  $.55 \leq \alpha_1 < .65$ ,  $.49 \leq \alpha_2 \leq .5$ ,  $3.4 \leq \alpha_3 \leq 4.1$ ,  $.04 \leq W_0 \leq .057$ ,  $.01 \leq W_1 \leq .38$ ,  $.3 \leq W_2 \leq .57$ ,  $.018 \leq Z_2 \leq .027$ ,  $.74 \leq \beta_L \leq .8$ ,  $.65 \leq \beta_1 \leq .82$ , and  $.0143 \leq \beta_{FK} \leq .055$ . Despite this narrow range, the parameter band for other types of volatile behavior is wide. In addition, the sensitivity analysis for the  $W_1$  parameter indicates that a second chaotic parameter space is likely to exist.

## VII. Conclusion

This paper develops a three-class predator-prey model for the behavior of the macro economy. The nexus of class relations between labor, industrial capitalists and financial capitalists captures important struggles over the distribution of income observed during the neoliberal era. Financial capitalists are added to the typical capital-labor interaction and are modelled as super-predators. In this role, financial capitalists are shown to be responsible for a finance capitalist-induced profit squeeze.

The basic model employed is a three-differential equation model. The unsustainable nature of neoliberal macro dynamics that resulted in the Great Recession is captured by the basic model. A key result is that unregulated finance capital has devastating impacts on the economy – a financial profit squeeze crash. This result is quite plausible in a three species/class model, as an

unregulated super-predator can easily mismanage the population of the other two classes. I refer to this as the FK paradox.

An extended model that incorporates bounding mechanisms, most importantly wealth effects capable of sustaining consumption behavior during significant reductions in labor share of income, shows that a continuum of undesirable behavior from chaotic bursts to irregular limit cycles to damped cycles often at depressed levels of economic activity characterize the behavior of the economy. Even the more desirable of these outcomes are propped up by unsustainable consumption demand from pseudo asset-price bubble-induced wealth effects.

The main conclusions of the study are that increases in the relative and/or absolute power of financial capitalists lead to increases instability, unregulated finance underlies the financial capitalist paradox, and in general unbalanced power relations among the three classes, depending on its particular nexus, results in various crisis generating mechanisms ranging from financial capitalist, reserve army or double profit squeezes to under-consumption problems.

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## Appendix A: Variable Definitions and Data Sources

### A. Stylize Facts

$\mu$ : Labor share of income.

Shares of gross domestic income: Compensation (pay and benefits) of employees, paid

Percent, Annual, Not Seasonally Adjusted

Linearly interpolated to quarterly data

U.S. Bureau of Economic Analysis, Shares of gross domestic income: Compensation of employees, paid [A4002E1A156NBEA], retrieved from FRED, Federal Reserve Bank of St. Louis; <https://fred.stlouisfed.org/series/A4002E1A156NBEA>, December 10, 2019.

(1- $\mu$ ): Profit share of income . NFC Profit Share Variables

All variants of the profit share are constructed from NIPA Table 1.14 with the exception of the net new equity issues adjustment which comes from Flow of Funds data table F.102, line 39. NIPA line numbers appear in parentheses, while the F.102 line numbers are in square brackets.

(1- $\mu$ )= net operating surplus (24) + taxes on production & imports (less subsidies) (23)  
net value added (19).

(1- $\mu$ -f): net profit share= (1-u) – net dividends (30) + net new equity issues [39]  
net value added (19)

Consumption share of income

Personal Consumption Expenditures:

Nominal Consumption: Personal Consumption Expenditures

Billions of dollars, seasonally adjusted.

U.S. Bureau of Economic Analysis, Personal Consumption Expenditures [PCE], retrieved from FRED, Federal Reserve Bank of St. Louis; <https://fred.stlouisfed.org/series/PCE>, December 6, 2019.

C: Real personal consumption expenditures:

C= RPCE

Real Personal Consumption Expenditures

Source: U.S. Bureau of Economic Analysis

Federal Reserve Economic Data - <https://fred.stlouisfed.org/series/PCECC96>

Billions of Chained 2012 Dollars

Quarterly, Seasonally Adjusted Annual Rate

C Share: C/RGDP, where

Y: RGDP

Real Gross Domestic Product

Billions of Chained 2012 Dollars

Quarterly, Seasonally Adjusted Annual Rate

Source: U.S. Bureau of Economic Analysis

<https://fred.stlouisfed.org/series/GDPC1>

W: Real US Wealth = (USWEALTH/PCECTPI)\* 100

USWEALTH :All sectors; U.S. wealth, Level

Millions of Dollars – converted to billions

Quarterly, Not Seasonally Adjusted

Source: Board of Governors of the Federal Reserve System (US)

<https://fred.stlouisfed.org/series/BOGZ1FL892090005Q>

I: Real investment: (NFCGFI/IDEFLATOR)\* 100

Nonfinancial business; gross fixed investment, nonresidential structures, equipment, and intellectual property products, Flow

Millions of Dollars – converted to billions of dollars

Quarterly, Not Seasonally Adjusted

Source: Board of Governors of the Federal Reserve System (US)

<https://fred.stlouisfed.org/series/BOGZ1FA145013005Q>

IDEFLATOR

Investment deflator

Real private fixed investment: Nonresidential (chain-type price index)

Index 2012=100, Quarterly, Seasonally Adjusted

<https://fred.stlouisfed.org/series/B008RG3Q086SBEA>

C-I RATIO

Real personal consumption expenditure/real nonfinancial corporate gross fixed investment

## B. Econometric Estimation of Behavioral Relations

$\mu$ ,  $1-\mu$ ,  $1-\mu-f$ , W, C, I, Y as defined above.

v: Degree of full employment: 1-UN3 or 1-UN6 where

UN3

Unemployment Rate U3 - Civilian Unemployment Rate

Source: U.S. Department of Labor: Bureau of Labor Statistics

Release: Employment Situation

Seasonally Adjusted, Quarterly

Aggregation Method: Average

Units: Percent – changed to decimal

Notes: Persons 16 years of age and older.

Limitations: only until 2011, seasonally adjusted

UN6

Unemployment Rate U6 - Total unemployed, plus all marginally attached workers plus total employed part time for economic reasons

Source: U.S. Bureau of Labor Statistics

Federal Reserve Economic Data - <https://fred.stlouisfed.org/series/U6RATE>

Percent, Monthly - converted to quarterly (average)

Seasonally Adjusted, monthly

RBAA: BAA – percentage change in IDEFLATOR

BAA

Title: Moody's Seasoned Baa Corporate Bond Yield

Source: Board of Governors of the Federal Reserve System

Seasonal Adjustment: Not Applicable

Frequency: Quarterly

Aggregation Method: Average

Units: Percent

RPRIME: MPRIME – percentage change in PCECPTPI (below)

MPRIME

Title: Bank Prime Loan Rate

Source: Board of Governors of the Federal Reserve System

Units: Percent

Frequency: Monthly – Converted to quarterly (average)

Seasonal Adjustment: Not Applicable

PCECTPI

Personal Consumption Expenditures: Chain-type Price Index

Index 2012=100, Quarterly, Seasonally Adjusted

Source: U.S. Bureau of Economic Analysis

<https://fred.stlouisfed.org/series/PCECTPI>

CPI

Consumer Price Index: All Items Excluding Food and Energy for the United States,

Index 2015=100,

Quarterly, Not Seasonally Adjusted

Source: Organization for Economic Co-operation and Development

Federal Reserve Economic Data - <https://fred.stlouisfed.org/series/USACPICORQINMEI>

RNFCDEBT: (NFCDEBT/IDEFLATOR)\* 100

NFCDEBT

Nonfinancial corporate business; debt securities and loans; liability, Level

Billions of Dollars, Quarterly, Seasonally Adjusted

Source: Board of Governors of the Federal Reserve System (US)

<https://fred.stlouisfed.org/series/BCNSDODNS>

RCONCR: (CONCR/ PCECTPI)\* 100

CONCR

Households and nonprofit organizations; consumer credit; liability, Level

Billions of Dollars, Quarterly, Seasonally Adjusted

Source: Board of Governors of the Federal Reserve System (US)

<https://fred.stlouisfed.org/series/HCCSDODNS>

PCFINOP: Percentage change in FINOP.

FINOP: Chinn and Ito (2008); p.31 develop an annual financial openness index ( an inverse measure of the extent of capital controls) that “codify the tabulations of restrictions on cross-border financial transactions reported in the IMF’s annual report on exchange arrangements and exchange rates, restrictions on both current and capital account transactions, and the required surrender of export proceeds. The index is calculated for a subset of 20 industrialized countries contained in the panel data set for 181 countries. The data is available at [http://web.pdx.edu/~ito/Chinn-Ito website.htm] .

The index for industrialized countries is constructed as described in Bezreh and Goldstein (2013; pp. 22-23.

STOCKPR

Total share prices for all shares for the U. S.

Organization for Economic Co-operation and Development, Total Share Prices for All Shares for the United States [SPASTT01USM661N], retrieved from FRED, Federal Reserve Bank of St. Louis; <https://fred.stlouisfed.org/series/SPASTT01USM661N>, December 15, 2019.

PCSTOCKPR

Calculated as the percentage change of the above index.

## Appendix B. Econometric Estimation of Model Parameters

The parameters of behavioral equations in the bounded model are estimated using a quasi-first difference (Cochrane-Orcutt) regression method of correct for autocorrelated residuals. In addition, robust standard errors are used for inferences. No corrections for endogeneity or non-stationarity with exception of including a deterministic linear time trend when relevant are applied.

Differential equations (1, 2', and 7) are estimated using the percentage change respectively in  $\mu$ ,  $f$  and  $W$  as the dependent variables. Given that the only parameter in the DEQ for  $v$  is endogenous, this equation does not require estimation. Independent variables are made up of the RHS variables in the three equations and additional controls when relevant and available. Independent variables for the three DEQs are respectively  $(f, v)$ ,  $((1-\mu), (1-\mu)^2, v, PCFINOP, PCSTOCKPR, TIME)$ , and  $(f, f^2, \mu Y/W, (\mu^*-\mu)Y/W, RBAA, PCSTOCKPR)$  where  $PCFINOP$  is the percentage change in an index of financial openness (liberalization) in industrialized nations,  $PCSTOCKPR$  is the percentage change in US stock prices and  $RBAA$  is the real yield on BAA bonds. All variable definitions and data sources are in Appendix A.

The behavioral equations for  $C$  and  $I$  (equations (4 and 5)), are estimated using respectively the following RHS variables:  $((1-\mu)Y, (\mu^*-\mu)Y, W, fY, TIME, RPRIME, RCONCR)$  and  $((1-\mu-f)Y, W, RBAA, RNFCDEBT, TIME)$  where  $RPRIME$  is the prime rate,  $RCONCR$  is real consumer credit and  $RNFCDEBT$  is NFC real debt. To meet the parameter restriction  $Z_1 = 1$ , the dependent variable used in the  $I$  equation is  $I-((1-\mu-f)Y)$ .

Econometric results are reported in Table B.1 only for parameters associated with the model. Ninety-five percent confidence intervals are reported for each parameter. Asterisks denote when parameters are two-tail and one-tail significant respectively as  $(**)$  and  $(*)$ . It should be noted that for variables that are one-tail significant, intervals with contain positive and negative values despite being significant.

Given the artificial use of quadratic components in the  $f$  and  $W$  DEQs to capture what is essentially discontinuous behavior in a continuous model, the estimation of quadratic parameters focuses more on realistic critical values of  $1-\mu$  and  $f$  where respectively  $\partial f/\partial(1-\mu) = 0$  and  $\partial W/\partial f = 0$  than the varied combination of

parameter values consistent with those critical values. As a result, I report confidence intervals for the critical values determined by the nonlinear combination of quadratic parameters that meet the above derivative conditions. Confidence intervals are determined using the Delta Method. Individual parameter values in these cases are not reported.

In addition, the  $\beta_0$  and  $Z_0$  parameters are employed to create an initial environment where  $E \ll Y$ . Thus, parameter value assignment in these cases may not be contained in estimated intervals.

Quarterly data is used over the period 1980:1 to 2010:4. Various subsamples are employed that eliminate the early 1980 recession and/or the Great Recession. The particular sample employed in each case is reported in Table B.1.

The results in Table B.1. show that the critical values associated with the  $f$  and  $W$  DEQs are realistic. In particular, the point estimate for the critical  $f$  value in the  $W$  equation is .0358 with a maximum value of .0694 implying that once  $f$  reaches these values, the  $f$  effect on  $W$  supposedly through asset prices turns negative. In comparison, the implied critical  $f$  value for FK PS Case 1 in Table 5 is .06. The critical  $(1-\mu)$  value in the estimated  $f$  DEQ has a point estimate of .321 and a maximum value of .614. In comparison, the critical  $(1-\mu)$  value for the same FK PS case is .20 which is contained within the confidence interval.

One issue with the econometric results is that despite key parameters being significant, estimates are not precise. Even in significant cases, confidence intervals are large. In addition, the  $f$  DEQ estimates with the exception of the critical  $(1-\mu)$  value are insignificant and intervals are even larger. Also, estimates of the  $W$  effect in  $C$  are on the low side. This is likely the result of not capturing the indirect effects of  $W$  on debt acquisition and thus  $C$ .



Table B.1.

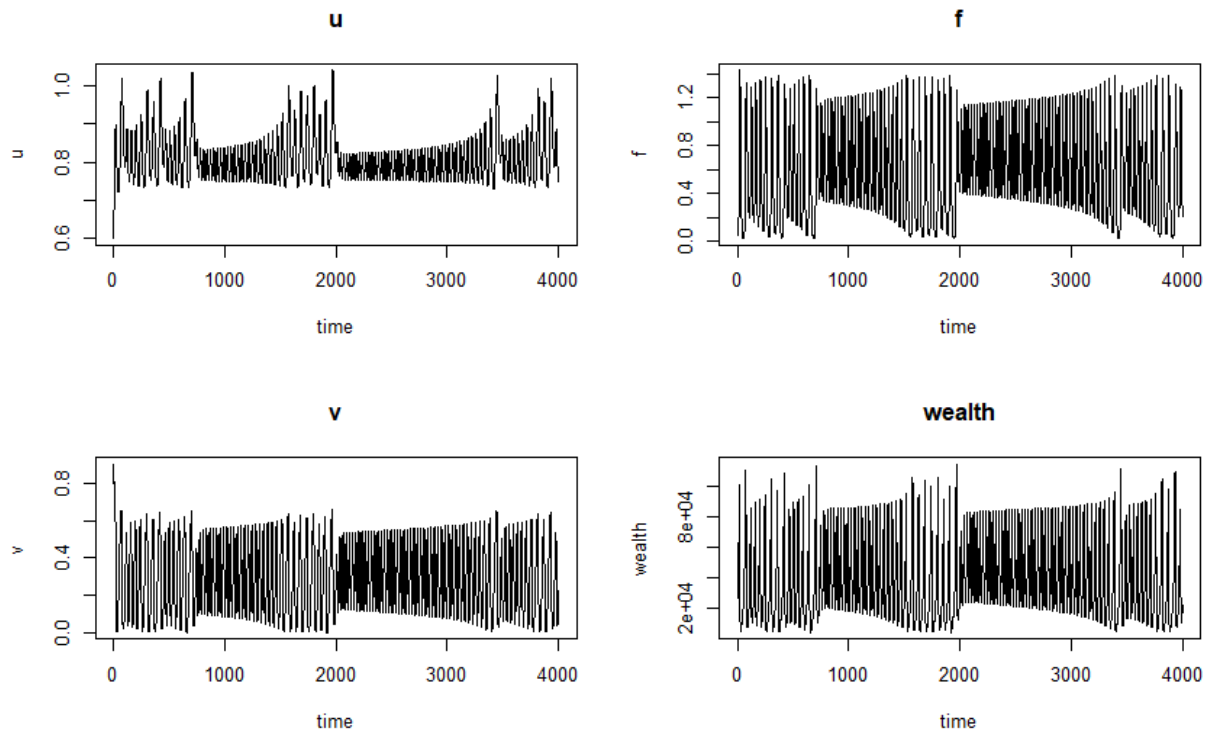
Eq./ Sample	$\gamma$	$\rho_1$	$\rho_2$	$\alpha_0$	$\alpha_2$	$(1-\mu)^*$	$Z_0$	$Z_1$	$Z_2$	$\beta_0$	$\beta_L$	$\beta_1$	$\beta_2$	$\beta_{FK}$	$W_0$	$f^*$
$\mu$ DEQ 1984:1 2010:4	-.161 to -.008 **	.008 to .171 **	-.019 to .013													
f DEQ 1980:1 2007:4 w/ TIME				-12.6 to 13.4	-10.2 to 3.34	.029 to .614 **										
W DEQ 1980:1 2007:4 w/ TIME															.018 to .150 **	.0022 to .069 **
C 1984:1 2007:4										61.3 to 858.4 **	.669 to .909 **	.404 to 1.15 **	.0002 to .0113 **			
l 1984:1 2010:4 w/TIME							-3157.0 to -482 **	1 Restrict -ed	.001 to .070 **							

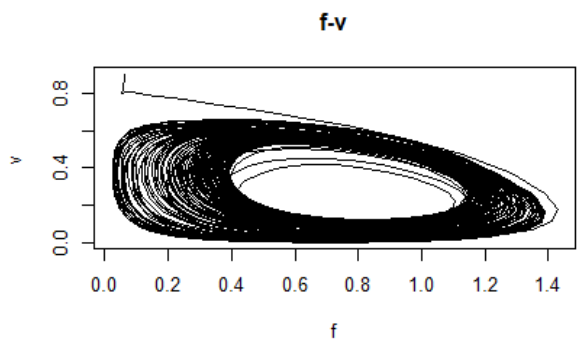
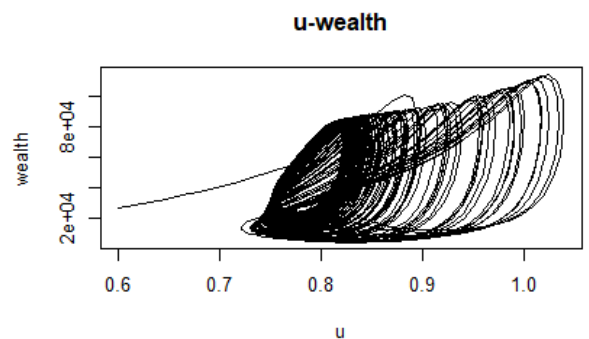
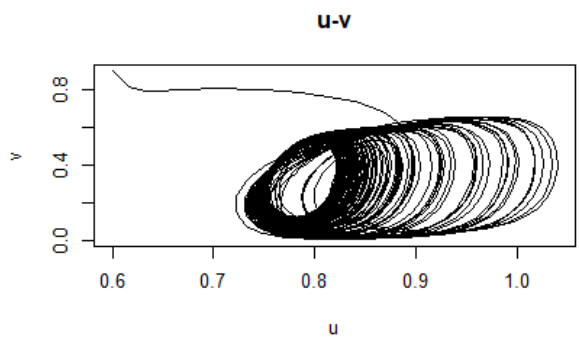
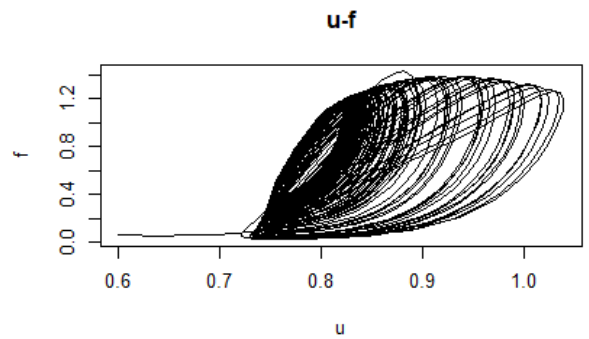
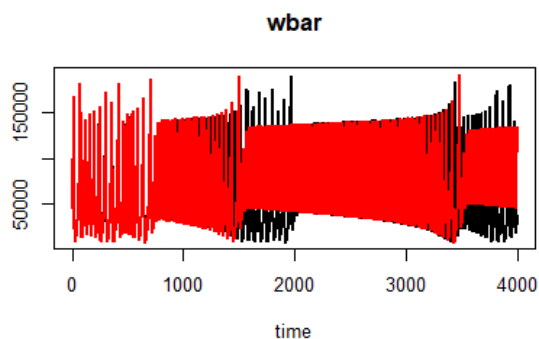
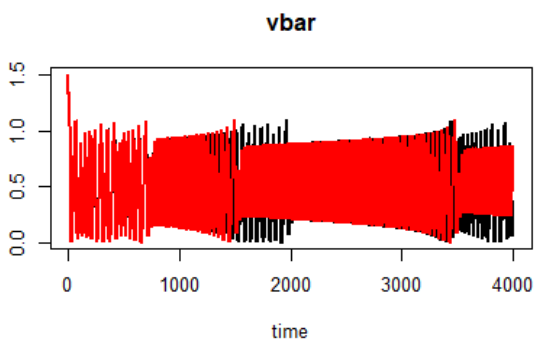
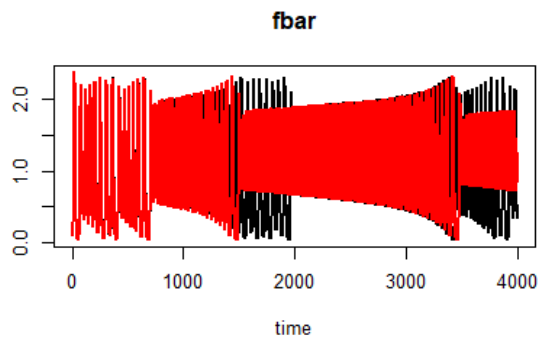
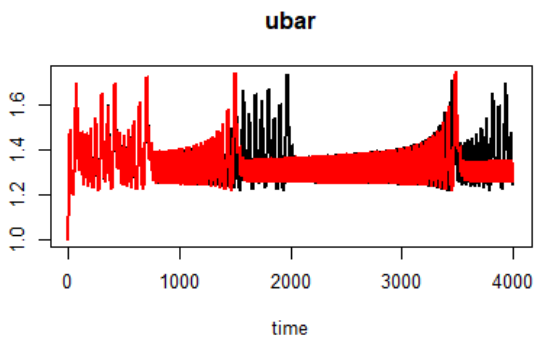
Appendix C. FK-RA PS detailed analysis of Case 6 Figure 3.

This appendix presents a more detailed analysis of the double profit squeeze simulation found in Figure 3, Case 6. The details presented here demonstrate model predictions contained in section 5.b. On the technical aspects of the model with the exception of a depiction of the fast and slow cycles which are evident from the state trajectories in Figure 3, Cases 2-4 and to a lesser extent Case 6. One detail to notice in the depiction of slow and fast cycles in Fig. 3 is as the fast cycle amplitude declines the slow cycle amplitude increases. This result is a precursor of a chaotic outcome.

The first set of graphs repeats the state trajectories depicted in Fig. 3, Case 6. The second set of graphs demonstrates how the trajectories are sensitive to changes in initial conditions. It should be noted that state variables followed by “bar” in graph titles refer original variables normalized by  $\mu(0)=.6$ . Thus  $\bar{v}=v/.6$ ,  $\bar{w}=\text{wealth}/.6$  etc. The original trajectories are black and the trajectories associated with a unit change in the fifth decimal place of the four state variable initial conditions appear in red.

The third set of graphs are a sampling of the two-dimensional phase-plane plots associated with the four state variables. As described in section 5.b, phase plots associated with two states within the same sub-system (nonlinear oscillator) exhibit limit cycle trajectories that either increase or decrease in amplitude as both states rise. For a state variable pair that crosses between sub-systems, in addition to changing amplitudes there appears a sharp reversal path that eventually ends up at a small amplitude limit cycle trajectory. This reversal occurs when the boundaries of state space are reached and the direct path back represents a dramatic change in behavior that can produce unexpected behavior.





Appendix D. Sensitivity Analysis Results

Table D.1. Sensitivity Analysis Results: FK PS Model

FKPS PARAMETER	QUALITATIVE BEHAVIOR BY PARAMETER RANGE
Base/Case 1	Damped Cycles (DC)
$\alpha_1$	$1.15 \leq \alpha_1 < 1.7$ DC MC. $(\mu, f, W) \uparrow$ . $.8 \leq \alpha_1 < 1.15$ LC-MI $(v, W)$ , $(\mu, f, W) \uparrow$ . $.3 \leq \alpha_1 < .8$ LC-IREG (ALL), $(\mu, f, W) \uparrow$ . $0 < \alpha_1 < .3$ LC-CB.
$\Upsilon$	$.025 < \Upsilon \leq .055$ DC-LESS $(\mu, f) \downarrow$ , $(v, W) \uparrow$ . $.055 \leq \Upsilon < .07$ MS-OOB. $\Upsilon \geq .07$ INT. $.007 \leq \Upsilon < .025$ DC-MC, $(\mu, f) \uparrow$ , $(v, W) \downarrow$ . $\Upsilon < .007$ OOB.
$\rho_1$	$.025 < \rho_1 \leq .038$ DC-MC $(f, \mu) \uparrow$ , $(v, W) \downarrow$ . $.038 < \rho_1 \leq .091$ LC $\rho_1 > .091$ M-MIX. $.01 \leq \rho_1 < .025$ DC-LESS. $\rho_1 < .01$ INT.
$\rho_2$	$.005 < \rho_2 \leq .26$ DC-LESS, $v \uparrow$ . $\rho_2 \geq .27$ INT
$\alpha_0$	$.1 < \alpha_0 \leq .42$ DC-MC, $(W, \mu) \uparrow$ . $\alpha_0 > .42$ M-MIX OOB. $0 < \alpha_0 < .1$ DC-LESS, $(W, \mu) \downarrow$
$\alpha_2$	$.25 < \alpha_2 \leq .5$ DC-LESS, $(\mu, f, W) \downarrow$ . $\alpha_2 > .5$ INT. $.15 < \alpha_2 < .25$ DC-MC, $.1 < \alpha_2 < .15$ LC, $.08 < \alpha_2 \leq .15$ LC-IREG. $.02 < \alpha_2 \leq .08$ INT. $\alpha_2 \leq .02$ M-MIX OOB
$\alpha_3$	$3.5 < \alpha_3 < 4.3$ DC-LESS, $(\mu, W, f) \downarrow$ . $2.2 < \alpha_3 < 3.5$ INT. $1.5 < \alpha_3 < 2.2$ MS-OOB. $4.3 < \alpha_3 \leq 5.3$ DC-MC, $(W, \mu) \uparrow$ . $5.3 < \alpha_3 \leq 5.7$ LC-MI $(W)$ . $5.7 < \alpha_3 \leq 6.2$ LC-IREG (ALL). $6.2 < \alpha_3 \leq 7.85$ LC-CB. $\alpha_3 > 7.85$ MIX CYCLES TO EQUIL.
$Z_1$	$1 \leq Z_1 < 1.6$ DC-LESS $f \uparrow$ , $W \downarrow$ . $Z_1 \geq 1.6$ MS $\mu, f$ , EX $v, W$ OOB. $.85 < Z_1 < 1$ DC-MC $W \uparrow$ , $.85 \leq Z_1 < .835$ LC-MI $W$ . $.835 \leq Z_1 < .8$ IREG CYC-EX OOB. $.8 < Z_1 \leq .6$ EX CYC. $Z_1 < .6$ IREG CYC&MS OOB
$Z_2$	$.02 \leq Z_2 \leq .54$ DC-MC, $Z_2 > .55$ MIX-SC OOB-TR. $0 < Z_2 < .02$ DC $v, W \uparrow$
$\beta_L$	$.75 \leq \beta_L < .88$ DC-LESS. $.88 \leq \beta_L \leq .99$ DC-IREG OOB-TR. $.54 \leq \beta_L < .75$ DC-MC. $.33 \leq \beta_L < .54$ LC-MI WIDE. $0 < \beta_L < .33$ LC-IREG. $\mu, f, W \uparrow$
$\beta_1$	$.75 < \beta_1 < 1$ DC-LESS $W \downarrow v \uparrow$ . $0 < \beta_1 < .75$ DC-MC $W \uparrow$
$\beta_{FK}$	$.02 \leq \beta_{FK} < 1$ DC-LESS $W \downarrow$ . $0 \leq \beta_{FK} < .02$ DC-MC
$W_0$	$.1 < W_0 < .25$ DC-MC $f, v, W \uparrow$ . $.25 \leq W_0 < .3$ LC $f, v, W \uparrow$ . $.3 \leq W_0 < .75$ LC-MI WIDE. $.75 \leq W_0 < .9$ LC-IREG CHAOS WIDE. $W_0 \geq .9$ MIX OOB-TR INT. $0 < W_0 < .1$ DC-LESS $f, W \downarrow$
$W_1$	ALREADY REPORTED IN TEXT.
$W_2$	$W_2 > 5$ DC-LESS $f, W \downarrow$ . $.8 < W_2 < 5$ DEC-MC $f, W, v \uparrow$ . $.5 \leq W_2 \leq .8$ LC $v, W \uparrow$ . $.25 < W_2 < .5$ LC-MI WIDE. $.17 \leq W_2 \leq .25$ CHAOS. $W_2 < .17$ MIX OOB-TR

Legend:

CB: Chaotic burst

DC: Damped cycle

EX: Explosive

INT: integration failure after relatively small number of periods IREG: Irregular/erratic behavior

LC: Limit cycle

LESS: Less cycles before equil.

MIX: mixture of stable and unstable

MC: more cycles before equil.

MI: Minor Irregularities

MREG: more regular

MS: monotonically stable

OOB: at least one state out of bounds.

TR: trivial equilibrium

WIDE: widespread behavior across state variables

$\uparrow$ : increases

$\downarrow$ : decreases

Table D.2. Sensitivity Analysis Results: FK-RA PS Model

FK-RA PS PARAMETER	QUALITATIVE BEHAVIOR BY PARAMETER RANGE
Base/Case 1	Chaotic oscillations
$\alpha_1$	$1.15 \leq \alpha_1 < 1.7$ DC MC. $(\mu, f, W) \uparrow$ . $.8 \leq \alpha_1 < 1.15$ LC-MI $(v, W)$ , $(\mu, f, W) \uparrow$ . $.3 \leq \alpha_1 < .8$ LC-IREG (ALL), $(\mu, f, W) \uparrow$ . $0 < \alpha_1 < .3$ LC-CB.
$\Upsilon$	$.0063 \leq \Upsilon \leq .0073$ . Chaos. $.0074 \leq \Upsilon < .008$ . MIX. $.009 \leq \Upsilon \leq .04$ DC-LESS. $.05 \leq \Upsilon \leq .12$ MS-OOB. $\Upsilon > .12$ MIX INT OOB.
$\rho_1$	$.04 \leq \rho_1 \leq .049$ CHAOS. $.05 < \rho_1 < .08$ IREG CYC TO EQ. $\rho_1 \geq .08$ MIX OOB-TR. $.039 \leq \rho_1 < .04$ DC TO LC. $.02 \leq \rho_1 < .039$ DC-LESS. $.007 \leq \rho_1 < .02$ DC-LESS OOB. $\rho_1 < .007$ MS OOB
$\rho_2$	$.007 \leq \rho_2 \leq .009$ CHAOS. $.01 \leq \rho_2 \leq .06$ DC-LESS. $.06 < \rho_2 < .1$ DC-LESS OOB. $.1 \leq \rho_2 \leq .4$ MS OOB. $\rho_2 > .4$ INT OOB. $.005 \leq \rho_2 < .007$ DC-MC OOB. $\rho_2 < .005$ MIX OOB-TR
$\alpha_0$	$.097 \leq \alpha_0 \leq .13$ CHAOS. $.13 < \alpha_0 < .19$ DC-LESS OOB. $\alpha_0 > .19$ MS or MIX OOB. $.097 < \alpha_0 \leq .09$ IREG CYC & MIX. $.09 < \alpha_0 < .01$ MIX. $\alpha_0 < .01$ INT
$\alpha_1$	$.565 \leq \alpha_1 < .64$ CHAOS. $.65 \leq \alpha_1 \leq .8$ MIX. $\alpha_1 > .8$ INT. $.565 < \alpha_1 \leq .07$ IREG LC to LC to OOB. $0 \leq \alpha_1 < .07$ IREG CYC or MIX CYC.
$\alpha_2$	$.49 \leq \alpha_2 \leq .5$ CHAOS. $.5 < \alpha_2 \leq 1.1$ DC-LESS. $1.1 \alpha_2 < 3$ MS. $\alpha_2 > 3$ INT, $\alpha_2 < .49$ MIX OOB.
$\alpha_3$	$3.4 < \alpha_3 < 4.1$ CHAOS. $4.1 < \alpha_3 < 5.5$ MIX OOB. $5.5 < \alpha_3 < 6.5$ MIX w/ CYC. $\alpha_3 > 6.5$ MON MIX. $\alpha_3 \leq 3.4$ DC-LESS.
$Z_1$	$.99 \leq Z_1 \leq 1$ . CHAOS. $1 < Z_1 \leq 1.5$ DC-LESS. $.75 < Z_1 < .99$ MIX OOB. $Z_1 < .75$ INT.
$Z_2$	$.018 \leq Z_2 \leq .027$ CHAOS. $Z_2 > .027$ MIX. $0.006 \leq Z_2 < .018$ LC. $Z_2 < .006$ DC-MC.
$\beta_L$	$.74 \leq \beta_L \leq .8$ CHAOS to IREG LC. $.8 < \beta_L \leq .99$ DC-LESS. $.65 \leq \beta_L < .73$ MIX OOB. $\beta_L < .65$ INT
$\beta_1$	$.7 < \beta_1 \leq .85$ CHAOS. $.86 < \beta_1 < .99$ MIX. $.5 \leq \beta_L < .7$ LC. $\beta_L < .5$ DC-LESS.
$\beta_{FK}$	$.0001 \leq \beta_{FK} < .06$ CHAOS. $.06 < \beta_{FK} < .25$ MIX. $\beta_{FK} > .25$ INT
$W_0$	$.04 < W_0 < .055$ CHAOS. $.055 < W_0 < .14$ MIX. $W_0 \geq .14$ INT. $.02 \leq W_0 < .045$ LC-IREG to LC. $W_0 < .02$ DC-LESS.
$W_1$	$.33 \leq W_1 \leq .38$ CHAOS. $.39 \leq W_1 \leq .49$ MIX-OOB. $.03 < W_1 \leq .33$ IREG LC to LC to IREG LC. $.02 < W_1 \leq .01$ CHAOS.
$W_2$	$.4 \leq W_2 \leq .57$ CHAOS. $W_2 > .57$ MIX. $.2 < W_2 \leq .4$ IREG LC to LC. $.1 < W_2 < .2$ MIX. $W_2 < .1$ INT-OOB.