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Abstract

This paper offers a unified analytical treatment of Marx's theory of rent. I highlight the key role played by the price of the agricultural commodity in determining rent. I offer two closures of the model. The first closure is an elaboration of Marx's argument in Volume III of *Capital*. The second closure explicitly allows for the role of demand. I also show that total rent can be decomposed into three components: absolute rent, differential rent I, and differential rent II. A Marxist theory can explain rent in any system of capitalist commodity production which uses privately owned nonreproducible resources.

JEL Codes: B51

Keywords: rent, surplus value, Marx.

1 Introduction

In the discipline of economics, there are at least two different conceptions of rent (Brown, 1941; Wessel, 1967). The first conception comes from the work of Vilfredo Pareto and conceptualises rent as the excess return to a factor of production over its next best alternative

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use (Wessel, 1967). This notion of rent is the dominant way of conceptualizing 'economic rent' in contemporary economics. It has been used in the influential literature on rent-seeking activities and its negative impact on economic growth and efficiency (Krueger, 1974; Murphy et al., 1993).

There is an alternative and older conception of rent that comes from the work of classical thinkers like David Ricardo and Karl Marx. In this alternative understanding, rent is conceptualized as the fraction of national income appropriated by owners of scarce, nonreproducible resources that can be used in the process of production. Here the focus is explicitly on the distribution of income between the fundamental classes of society. A neoclassical interpretation of this alternative, classical conception of rent can replace social classes with factors of production and thinks of rent as the excess return earned by a factor of production over what would be necessary to induce it to do its work (Wessel, 1967). In this paper, I will work with the classical conception of rent as a fraction of national income and investigate the economic mechanisms that can account for its emergence. In particular, I will develop a unified, analytical account of the classical theory of rent presented in Volume III of *Capital* (Marx, 1993).

In the three volumes of *Capital*, Marx offers a penetrating, critical analysis of the structure and long term dynamics of the capitalist mode of production. The analysis and presentation in *Capital* is organized into three volumes and conducted at two different levels of abstraction. Volumes I and II operate at the level of what Marx calls 'capital in general', where competition between capitals is abstracted from, and Volume III operates at the level of 'many capitals', where competition is introduced back into the analysis. In terms of substantive issues, in Volume I of *Capital*, Marx analyses the process of production of capital, i.e. the process of the generation of surplus value through the exploitation of labour, and the accumulation of surplus value to create additional capital.[1] After analysing the issues related to the realisation of surplus value in Volume II, Marx takes up the analysis of distribution of surplus value in Volume III of *Capital*.[2]

[1]Towards the end of Volume I, Marx introduces the issue of distribution of surplus value. "The capitalist who produces surplus-value, i.e. who extracts unpaid labour directly from the workers and fixes it in commodities, is admittedly the first appropriator of his surplus-value, but he is by no means its ultimate proprietor. He has to share it afterwards with capitalists who fulfil other functions in social reproduction taken as a whole, with the owner of the land, and with yet other people. Surplus-value is therefore split up into various parts. Its fragments fall to various categories of person, and take on various mutually independent forms, such as profit, interest, gains made through trade, ground rent, etc." (Marx, 1992, 709). But the full-blown analysis of distribution has to wait till Volume III, when the analysis moves to a lower level of abstraction.

[2]For a discussion of the structure and content of *Capital*, see Basu (2017).

Marx's analysis of the distribution of surplus value in Volume III proceeds in two analytically separate steps. In the first step, the total surplus value generated in production is distributed across different sectors through the competition between 'industrial' capitals.[3] Competition is manifested in the mobility of capital across sectors in search of higher rates of profit. The long run equilibrium of this competitive process is the emergence of an average rate of profit across all sectors, supported by 'prices of production', which necessarily implies a redistribution of the different magnitudes of surplus value generated across sectors due to the differences in organic compositions of capital (Baumol, 1974). In the second step, some of the surplus value appropriated by 'industrial' capital is further redistributed to other fractions of the ruling class as commercial profit, ground-rent and interest.

While Marx was clear that ground-rent could arise in any sector that used natural resources for commodity production, he offered most of his analyses in terms of the specifics of agricultural production. The emergence of ground-rent in agriculture requires, according to Marx, two conditions: a lower than average organic composition of capital in agriculture, and the presence of landed property, where with landed property comes the monopoly of use of the land.

> Landed property presupposes that certain persons enjoy the monopoly of disposing of particular portions of the globe as exclusive spheres of their private will to the exclusion of others. (Marx, 1993, 752).

The first condition - lower than average organic composition of capital in agriculture - gives rise to higher than the economy-wide average rate of profit. Marx refers to this as 'surplus profit'. The second condition - presence of landed property - erects what Marx calls an 'absolute barrier' to the mobility of capital. The impediment to the free movement of capital into agriculture has two effects: it prevents the surplus profit from flowing away from agriculture, as would have happened if there were no barriers to the mobility of capital into agriculture; and, equally importantly, it allows this surplus profit to be appropriated by the class of landowners, i.e. the owners of the land, as ground-rent.

Marx's theory of ground-rent outlined above raises two questions. The first question relates to the domain of applicability of the theory. Is Marx's theory of ground-rent applicable

[3]The term 'industrial capital' should be understood broadly as referring to capital that is involved in the production of commodities, which can be goods or services. It does not refer to industrial production only. Marx uses this term to distinguish capital involved in production from 'merchant capital', which is involved in the purchase and sale of commodities and 'usurious capital', which is involved in lending and borrowing of money. Both these forms of capital predate 'industrial capital' and are characterized by the fact that they appropriate value through unequal exchange but do not organize the production of commodities and the concomitant generation of surplus value.

beyond agriculture? The answer is in the affirmative, as Marx himself emphasised. The theory of ground-rent can be applied to any sector of capitalist commodity production which uses a nonreprdoducible natural resource that is privately owned. Hence, the analysis can be applied to other sectors like mining, oil and natural gas, real estate, tourism. But this immediately raises the second question. How important is the assumption of lower-than-average organic composition of capital? If the assumption of lower-than-average organic composition of capital is crucial for the logic of emergence of ground-rent, then it is doubtful if Marx's theory can be applied to any sector at all. Not only is mining, oil and natural gas, and real estate as capital intensive as the rest of the economy, it is no longer possible to assert that even agriculture is less capital intensive than the rest of the economy (other than in some developing countries).

The main contribution of this paper is to develop a consistent, unified Marxist theory of rent.[4] In developing this theory, I will use the term "land" for any nonreprdoducible natural resource that is privately owned and can be used in the production of commodities; I will use the term "agriculture" to refer to the economic activity of producing some commodity on the "land"; and I will use the term "agricultural commodity" to refer to the commodity that is produced on the "land". After setting up the basic model, I will highlight the key role played by the price of the agricultural commodity for the theory of rent.[5] I offer two mechanisms for determining the price of the agricultural commodity.

The first mechanism develops the first closure of the basic model of rent. In terms of specifics, this closure is a development of Marx's argument in Volume III of *Capital*. It relies on the juxtaposition of lower-than-average organic composition of capital and barriers to mobility of capital to explain rent. In developing this closure, I show that there is one significant way in which Marx's account needs to be amended: just lower than average organic composition of capital is not sufficient for generating rent; a stronger, concavity condition about the lower organic composition of capital in agriculture is needed.

The second mechanism develops the second closure of the model of rent. In this closure, I extend Marx's analysis in several significant ways. First, unlike the analysis in Volume III

[4]In this context, it is important to clarify that the commonly used term rent to refer to payments made to use a building, or apartment, for a fixed period of time is conceptually very different from 'rent' that I discuss in this paper. When a building or apartment is rented out, the transaction involves a piecemeal sale of a commodity (the building or the apartment). Thus the rent one pays to use a building or apartment for a fixed period of time is a portion of the price of the building or apartment. The 'rent' that I discuss in this paper, on the other hand, is a portion of the surplus value appropriated by an owner of a *non-produced* resource that is limited in quantity and can be used in capitalist commodity production.

[5]While Marx uses the term 'ground-rent', I will use the term 'rent' to emphasize its generality.

of *Capital*, this closure allows for a direct role of demand. Hence, unlike Marx's analysis and the contemporary neo-Ricardian analysis, where the output of the agricultural commodity is taken as given, I allow for the joint determination of the output and price of the agricultural commodity, the latter in turn, determining the magnitude of rent. Second, it moves away from the analysis of Volume III by dispensing with the assumption of lower-than-average organic composition of capital. This is very important because it now increases the applicability of this Marxist theory of rent to a wide range of contemporary contexts where the assumption of lower-than-average organic composition of capital can no longer be justified.

In developing the Marxist theory of rent, this paper connects with several strands of extant literature. First, it speaks directly to the large Marxist literature on rent (Eaton, 1963; Mandel, 1968; Fine, 1979; Rubin, 1979; Ghosh, 1985; Foley, 1986; Howard and King, 1992; Samuelson, 1992; Ramirez, 2009). The main contribution of this paper, with respect to the existing Marxist literature, is that I formalize the theory of rent. Marx used a series of, often confusing, examples in Volume III of *Capital* to illustrate his arguments. Later authors followed him in presenting arguments mostly with the help of examples. Specific examples are driven by specific assumptions, and so it is not possible to understand the general logic of the argument with the help of examples *only*. A formal, mathematical framework, on the other hand, can help us grasp general arguments.[6]

Second, it indirectly speaks to the contemporary neo-Ricardian literature on rent. Within the classical tradition, the theory of rent was elaborated most clearly by David Ricardo (Ricardo, 1821), which was further developed by Piero Sraffa (Sraffa, 1960). A clear, formal treatment of the neo-Ricardian theory of rent in a general n-sector economy with linear technology can be found in Kurz (1978). The Marxist theory of rent that I develop in this paper differs from the neo-Ricardian literature on rent in several ways. While the neo-Ricardian analysis uses a general equilibrium framework, the Marxian theory developed in this paper is a partial equilibrium analysis. This shortcoming of the theory developed in this paper comes with certain advantages over the the general equilibrium neo-Ricardian analysis. The neo-Ricardian analysis uses linear technology, but the Marxian theory allows for nonlinear technologies. The neo-Ricardian analysis does not have any place for demand, and conducts the whole analysis by taking the level of output (of the agricultural commodity)

[6]The discussion of rent in volume three of *Capital* shows that Marx was clearly aware of three types of ground-rent: (a) rent arising from monopoly price (Marx, 1993, pp. 898-899); (b) differential rent; and (c) absolute rent. While Marx briefly discusses the issue of rent that depends on monopoly price (for an illuminating example of a vineyard, see Marx (1993, pp. 910)), he devotes the bulk of the analysis to differential and absolute rent. In this paper, I follow Marx and only discuss differential and absolute rent.

as given. In contrast, the Marxian theory developed in this paper allows for the role of demand in jointly determining output and price of the agricultural commodity, the latter, in turn, determining the magnitude of rent. Finally, the neo-Ricardian analysis does not allow for the existence of absolute rent; the Marxian theory not only explains absolute rent, but accords it an important place in the whole analytical framework.[7]

Third, it connects to the emerging critical literature on land rent and urban housing (Haila, 2016; Obeng-Odoom, 2017; Ryan-Collins et al., 2017). While the traditional understanding of high rent and prices of real estate in urban areas has often seen zoning regulation as a key causative factor (Glaeser et al., 2005), the emerging critical literature digs deeper into the reasons for limited supply and tries to identify different classes that benefit from the restricted supply (Haila, 2016). This paper offers a coherent theoretical framework that could inform discussions of land rent in critical urban studies (Ball et al., 1985; Haila, 1990; Aalbers, 2016; Haila, 2016; Ward and Aalbers, 2016).

The rest of the paper is organized as follows. In section 2, I introduce the basic setup; in section 3, I define the terms 'surplus profit' and 'rent', discuss the three components of rent and highlight the key condition for the existence of rent; in section 4, I develop the first closure of the model of rent, which is an elaboration of the argument of Marx in Volume III of *Capital*; in section 5, I develop a second closure of the model that extends Marx's analysis by explicitly incorporating the role of demand and profit-maximizing behavior of capitalist farmers. I conclude the discussion in section 6 with some caveats and ideas about extensions.

2 Basic Set-up

Suppose total land is divided into N plots and is indexed by $i = 1, 2, \ldots, N$. Let A_i denote the area of plot i; and y_i denote the quantity of agricultural output produced on plot i.

2.1 Hierarchy of Quality and Rates of Profit

In this paper I will study a situation where technology and distribution is given, i.e. *the real wage rate is given* and each plot of land uses a *given technique of production.* Let c_i denote the constant capital (the amount of money used to purchase means of production) on plot i; and let v_i denote variable capital (the amount of money used to purchase labour power)

[7] One of the key insight emerging from Kurz (1978) is that rankings of plots of land and magnitudes of rent can change when there is a change in distributional factors. I incorporate this insight into my analysis by deriving all results on the basis of given technologies *and* real wage rates.

on plot i.[8] Thus, the total cost of production (which is also the total capital investment) on plot i is given by $c_i + v_i$, the sum of constant and variable capital.[9] Let k_i denote the average cost of production on plot i, i.e.,

$$k_i = \frac{c_i + v_i}{y_i}. \tag{1}$$

Let us define the relative "quality of a plot of land" with reference to its average cost of production, i.e. plot i is of higher quality than plot j if the average cost of production on the former is lower than in the latter: $k_i < k_j$.[10] Since the real wage rate and techniques of production on each plot are given, we can define the average cost of production on each plot and so, without loss of generality, let $i = 1, 2, \ldots, N$ index plots of land arranged in *increasing order of quality*.[11] Thus, plot 1 refers to the worst, plot 2 to the next-worst plot and so on, with plot N referring to the best plot of land.

The hierarchy of the quality of plots of land is an important characteristic of the agricultural economy under study and emphasizes the fact that the average cost of production stands in a relationship of hierarchy across plots, with the worst plot of land having the highest average cost and the best plot having the lowest average cost of production. We state this important characteristic of the agricultural economy as

Assumption 1. *Let $i = 1, 2, \ldots, N$ index the plots of land arranged in increasing order of quality, and therefore in decreasing order of average cost of production. Then, we have*

$$k_1 > k_2 > \cdots > k_N, \tag{2}$$

[8]The variable capital needs to be bounded away from zero to ensure that the organic composition of capital does not increase without bounds. The economic content of this assumption is that production cannot take place without labour, a most meaningful assumption. Moreover, I abstract from fixed capital in this paper. Hence, I assume that all the means of production is used up in one production cycle.

[9]All costs and prices are expressed in terms of units of money. One can use a monetary expression of labour time (MELT) to convert freely between monetary and value magnitudes, following the logic in Foley (1982). Using the MELT and taking an aggregate perspective allows us to deal with the so-called transformation problem that Samuelson (1971) thought was fatal to Marxian economics. For further details, see Foley (1982, 2000); Mohun and Veneziani (2017).

[10]Quality of a plot of land comes both from its "fertility" and "location" (Marx, 1993, pp. 789). The locational advantage of plots of land figures as an important determinant of rent and has been explored in the urban economics literature; for instance, see the classic account in Alonso (1964).

[11]If two or more plots have the same average average cost of production, we merge them into one and use their average value for the analysis in this paper.

where

$$k_i = \frac{c_i + v_i}{y_i}$$

denotes the average average cost of production on plot i.

The hierarchy of quality of plots of land will imply a reverse hierarchy of rates of profit when commodity production using land is organized along capitalist lines. We state this below as

Proposition 1. *Let r_i denote the rate of profit earned by capitalist commodity production on plot i, where $i = 1, 2, \ldots, N$. If the economy is characterised by a hierarchy in the quality of plots of land given by Assumption 1, then we have*

$$r_1 < r_2 < \cdots < r_N. \tag{3}$$

3 Surplus Profit and Rent

3.1 Definition and Logic

Let α denote the economy-wide average rate of profit, and let r_i denote the rate of profit on plot i. A comparison of r_i with α allows us to define what Marx referred to as 'surplus profit' on plot i as

$$SP_i = (c_i + v_i)r_i - (c_i + v_i)\alpha = (c_i + v_i)(r_i - \alpha), \tag{4}$$

where SP_i refers to surplus profit. Hence, surplus profit on any plot is the profit income earned by capital invested on that plot over and above what could be earned at the economy-wide average rate of profit.

Let us consider two different configuration of property relations under which agricultural production can be organized in this economy.

- The capitalist owner-farmer economy: In this configuration, all plots of land are owned by capitalist farmers, who organize agricultural production by hiring wage labour and investing their own (or borrowed) capital.

- The capitalist tenant-farmer economy: In this configuration, all plots of land are owned by non-cultivating landlords. Tenant-farmers organize agricultural production by hir-

ing wage labour and investing their own (or borrowed) capital, as before, but with the key difference that they have to rent out land – the key input into agricultural production – from the landlords.

If commodity production is organized under the first configuration of property relations, all surplus profit is earned by capitalist farmers as super-normal profit. On the other hand, if commodity production is organized under the second configuration of property relations, i.e. if we consider a capitalist tenant-farmer economy, then the surplus profit on plot i takes the form of rent. The landlord who owns plot i rents it out for a rental payment that is exactly equal to the surplus profit, and thereby appropriates the surplus profit as ground rent. This is the essence of Marx's idea that rent is merely a 'transformation of surplus profit'. Hence, letting GR_i denote the rent on plot i, we have

$$GR_i = SP_i = (c_i + v_i)(r_i - \alpha). \tag{5}$$

What is the logic for the emergence of rent? In a capitalist economy, land is privately owned - by the class of landowners. Hence there is no free land. Since land can be used in capitalist commodity production, it confers a strategic advantage to the owners of land. They can withhold access to land unless the capitalist commodity producer is willing to make a payment - the rental payment - for accessing the land. Hence, landowners can bargain with capitalist commodity producers who use land for a share of the surplus value they realize. One possible equilibrium outcome of the bargaining between landowners and capitalist commodity producers is that the rental payment is exactly equal to the surplus profit generated on a plot of land. If the capitalist gives up all of the surplus profit to the landowner as rent, she is left with exactly the amount of surplus value that ensures her an economy-wide average rate of profit. By moving her capital to a different line of production, she would earn, on average, exactly what she earns in capitalist commodity production using land. Hence, she has no incentive to deviate from the equilibrium outcome. The landowner has no incentive to deviate too. This is because she cannot expect to get a rental payment in excess of the surplus profit. If she asks for a rental payment that is larger than the surplus profit, the capitalist will move to another line of production. Hence, this is the best the landowner can do. Thus, the equilibrium outcome of the bargaining between landowners and capitalist commodity producers leads to a contractual rental payment from the capitalist to the landowner - which is rent - that is exactly equal to the surplus profit.

3.2 Components of Rent

Marx had argued that the total rent on any plot of land can be decomposed into three parts, differential rent of the first variety, differential rent of the second variety and absolute rent. A little algebraic manipulation shows that this is indeed the case:

$$GR_i = DRI_i + DRII_i + AR \tag{6}$$

where

$$DRI_i = (c_i + v_i)(r_i - r_1)$$

is differential rent of the first variety,

$$DRII_i = [(c_i + v_i) - (c_1 + v_1)]\,(r_1 - \alpha)$$

is differential rent of the second variety, and

$$AR = (c_1 + v_1)(r_1 - \alpha)$$

is absolute rent.

The logic underlying the three components of rent are very different. Differential rent of the first variety arises due to differences in the quality of the plots of land, as captured by the differences in the average cost of production across plots. For plot i, the benchmark plot for making quality comparisons is the marginal land, i.e. the worst plot of land. Hence,

$$DRI_i = (c_i + v_i)(r_i - r_1),$$

where $r_i - r_1$ captures the difference in quality of plot i with respect to the benchmark plot, and $c_i + v_i$ is the magnitude of capital invested on the plot. Since no production can take place without labour, the magnitude of variable capital on plot i is bounded away from zero (while the magnitude of constant capital is non-negative). Hence $c_i + v_i > 0$, which implies that $DRI_i \geq 0$ (since $r_i - r_1 > 0$ by Proposition 1).

Differential rent of the second variety arises from the differences in the magnitudes of capital invested across plots of land. For plot i, the difference in magnitude of capital invested

is also measured with respect to the marginal land, i.e. the worst plot of land. Hence,

$$DRII_i = [(c_i + v_i) - (c_1 + v_1)]\,(r_1 - \alpha),$$

where $[(c_i + v_i) - (c_1 + v_1)]$ captures the difference in magnitude of capital invested. This way of defining $DRII$ is different from the way Marx defined it. To define $DRII$, Marx abstracts from differences in quality of plots of land and thinks of it as the surplus profit on all 'doses of capital' in comparison to the least productive dose. The key difficulty in this definition is that there is no natural way to define 'doses of capital'.[12] I avoid this difficulty by defining $DRII$ as the part of rent coming from the difference in magnitude of capital invested vis-a-vis the worst plot.

Of course, this implies some ambiguity about the sign of the differential rent of the second variety. As long as the total capital invested on plot i is higher than the capital invested on the worst plot of land, $DRII$ will be positive. But, if the total capital invested on plot i were to fall below the capital investment on the worst plot, then the magnitude of the differential rent of the second variety will turn negative. This does not create any problems for the overall analysis because the sum total of DRI and $DRII$ will always be positive as long as the total rent on all plots is greater than the total rent on the worst plot.

While we can see easily that differential rent of both varieties arise from differences with respect to the worst plot of land - DRI from differences in quality, and $DRII$ from differences in capital invested - it also implies that the worst plot of land cannot generate these types of rent. Hence we have to confront the question as to whether the worst plot generates any rent. While Ricardo, and the neo-Ricardian tradition building on Sraffa's formalization of Ricardo, thinks that there can be no rent on the marginal land, Marx argues that position. If land has become private property and there is no free land, argues Marx, then no landowner, including the owner of the worst plot, will give her plot gratis.[13] In the Marxist framework, the total rent on the worst plot of land is called 'absolute rent'. That is why the absolute

[12] One way to circumvent this problem is to instead use the marginal cost of production, as I do in the second closure in section 5 below. The marginal cost of production is the "dose" of capital needed to produce the last unit of output on each plot of land. Hence it is a natural way to extend and complete Marx's idea about "dose of capital".

[13] As long as landed property exists, no plot of land, including the worst plot, will be available *gratis* for use in commodity production (Marx, 1993, pp. 884–885, 890–891). "Assuming then that demand requires the taking up of new land which is, say, less fertile than that previously cultivated, will the owner of this land lease it for nothing just because the market price of its product has risen high enough for capital investment to pay the farmer the price of production and thus yield him the customary profit? In no way. The capital investment must yield him a rent. He leases only when a lease-price can be paid." (Marx, 1993, pp. 891).

rent is given by

$$AR = (c_1 + v_1)(r_1 - \alpha),$$

which is the surplus profit earned on the worst plot of land. Since $r_i > \alpha$ for $i = 1, 2, \ldots, N$, this means that $r_1 - \alpha > 0$. Hence, absolute rent is positive.

3.3 Key Condition for the Emergence of Rent

What is the key condition for the generation of rent? Since rent is merely a transformation of surplus profit, the key condition for rent is the same as the condition for the existence of surplus profit. Proposition 1 shows that existence of surplus profit on the worst plot is sufficient to ensure existence of surplus profit on all plots. Hence, the key condition for the generation of rent on all plots is the existence of surplus profit on the worst plot of land.

Let $\tilde{p}$ denote the price of the agricultural commodity which ensures the economy-wide average rate of profit, α, *on the worst plot of land*, i.e. the plot of land indexed by $i = 1$. Since the output on plot 1 is given by y_1, we have

$$\tilde{p} y_1 = (c_1 + v_1)(1 + \alpha)$$

so that

$$\tilde{p} = \left(\frac{c_1 + v_1}{y_1} \right) (1 + \alpha) = k_1 (1 + \alpha). \tag{7}$$

Whenever the price of the agricultural commodity is higher than $\tilde{p}$, *all* plots of land will generate surplus profit. Let us note this as

Proposition 2. *Let p denote the market price of the commodity produced on land. If $p > \tilde{p}$, then all plots of land generate surplus profit and hence rent.*

The proof follows immediately from a conjunction of two facts: (a) that the price level of $\tilde{p}$ ensures surplus profit on the worst plot of land, and (b) that there is a hierarchy of profit rates in agriculture with profit rates increasing with the quality of the plot of land (see Proposition 1).

The generation of surplus profit and its transformation into rent on all plots of land is depicted in Figure 1 and 2. In each of these figures, the plots of land are arranged in increasing order of quality on the horizontal axis, and prices are measured on the vertical

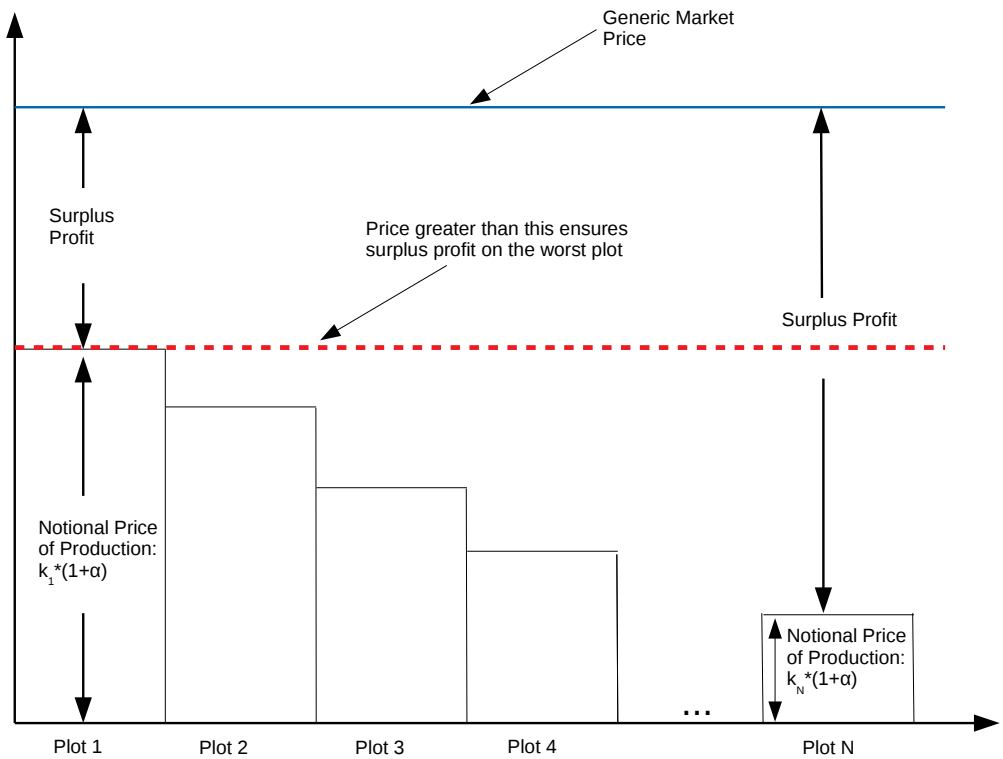

Figure 1: *Surplus profit on all plots of land with a relatively high market price of the commodity.*

axis. Two important price levels are marked on the figures: $\tilde{p}$ is shown as a (red) broken horizontal line, and the market price is shown as a (blue) solid horizontal line. The vertical boxes represents the notional price of production on each plot of land, i.e. it is the price which would ensure the economy-wide rate of profit on the plot. For instance on plot i, the height of the vertical box is given by $k_i(1+\alpha)$. The (red) broken horizontal line, which indicates the price $\tilde{p}$, is drawn exactly at the height of the vertical box for plot 1 (worst plot) because $\tilde{p}$ is the level of market price which would ensure the economy-wide average rate of profit on the worst plot.

The (blue) solid horizontal line represents the market price and the difference between the market price and the height of the vertical box on any plot gives the surplus profit on that plot. Thus, the surplus profit on the worst plot is the difference in the heights of the

(blue) solid horizontal line and the (red) broken horizontal line. On any other plot, the total surplus profit is higher in magnitude and includes the surplus profit on the worst plot as its part. Turning to Figure 2, we see the transformation of surplus profit into rent. On any plot, the total surplus profit is the total rent. The total rent on the worst plot is absolute rent. On any other plot, the total rent is the sum of absolute rent and differential rent.

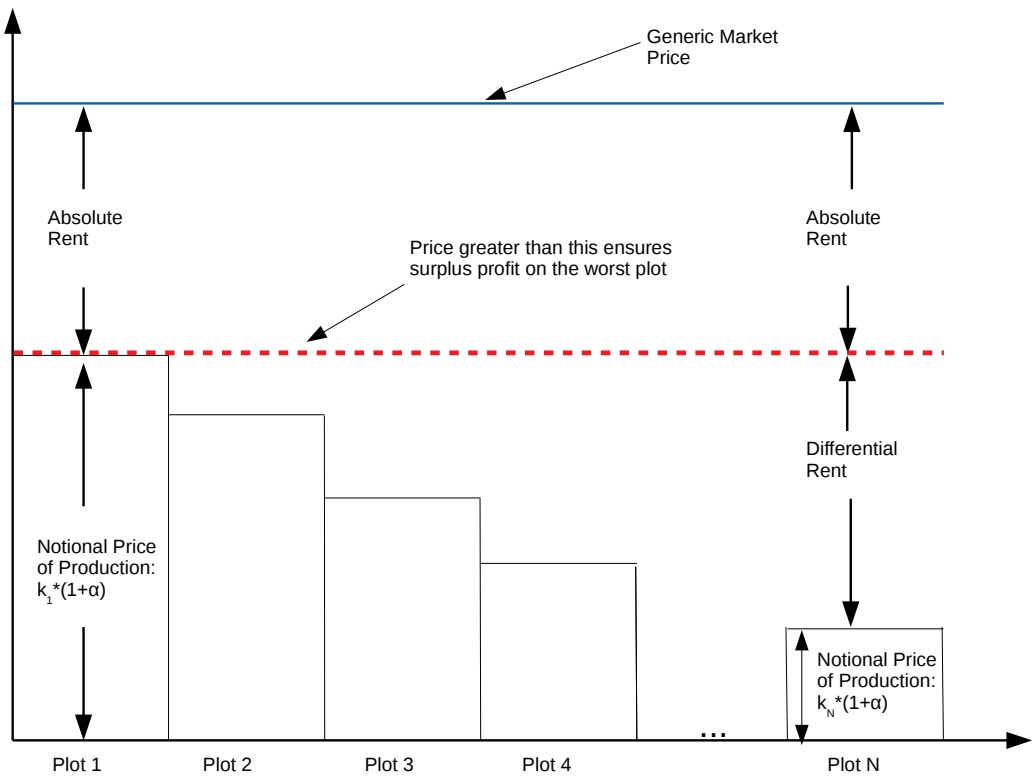

Figure 2: *Ground-rent on all plots of land with a relatively high market price of the commodity.*

The analysis in this section shows that the price of the (agricultural) commodity is key to the emergence of rent. We offer two closures of the above model, i.e. two mechanisms to determine the price of the agricultural commodity. The first closure is a formalization of Marx's ideas in Volume III of *Capital*. Here price is determined by the principle that all surplus value generated in agriculture is realized within agriculture because of barriers to entry of capital. If the organic composition of capital in agriculture is 'sufficiently lower'

than the economy-wide average organic composition, then zero net flow of surplus value can generate rent on all plots of land. The key problem of this closure is that it does not allow for the role of demand in the the determination of the price of the agricultural commodity. The second closure addresses this lacuna and explicitly allows for demand to jointly determine the output and price levels.

4 Determination of the Price Level: First Closure

In Marx's analysis the price of the agricultural commodity is determined by the principle that agriculture retains all the surplus value it generates. Let p^* denote the price of the agricultural commodity at which agriculture as a whole realises the total surplus value it generates. Let $Y = \sum_i y_i, C = \sum_i c_i$ and $V = \sum_i v_i$; then,

$$p^* Y = (C + V)(1 + r^*)$$

where

$$r^* = \frac{eV}{C + V} = \frac{e}{1 + (C/V)} = \frac{e}{1 + OCC_A} \tag{8}$$

and e is the *common* rate of exploitation (ratio of surplus value and variable capital) that obtains in all sectors of the economy, and

$$OCC_A = \frac{C}{V} = \frac{\sum_i c_i}{\sum_i v_i}$$

is the organic composition of capital for the agricultural sector as a whole. Hence,

$$p^* = \left(\frac{C + V}{Y}\right)(1 + r^*)\,. \tag{9}$$

The agricultural economy is characterised by the following two conditions, where the first condition relates to the price of the agricultural commodity and the second relates to the organic composition of capital in agriculture.

Assumption 2. *(Price and Organic Composition of Capital).*

1. *Price: Let the price of the agricultural commodity, p, be determined by the principle that there is zero net flow of surplus value from agriculture, so that $p = p^*$, with p^**

defined in (9).

2. *Organic composition of capital: Let* $z_1 = 1 + OCC_A$*, where* OCC_A *denotes the organic composition of capital in agriculture as a whole; let* $z_2 = 1 + OCC_E$*, where* OCC_E *denotes the economy-wide organic composition of capital. Let the two be related as follows:*

$$z_1 \leq f(z_2) = \frac{ez_2}{(\beta - 1) z_2 + e\beta},$$

where

$$\beta = \frac{(c_1 + v_1)/y_1}{(C + V)/Y}$$

and we have $1 + e > \beta$*, and* $z_2 > e\beta/(1 + e - \beta)$*.*

The first part of Assumption 2 relates to the determination of the market price of the agricultural commodity. It stipulates that the market price of the agricultural commodity is determined by the principle that agriculture retains all the surplus value it generates. The latter assumption is justified by the fact of existence of barriers to movement of capital in agriculture. This implies that the price of the agricultural commodity is given by $p = p^*$, defined in (9). It is important to point out that this is only one, though by no means the only, way to determine the market price of the agricultural commodity. We use it in the first closure of the model because this is close to the intuitive idea Marx worked with: barriers to the movement of capital into agriculture would allow agriculture to retain the surplus profit (and hence the total surplus value) it generates.

The second part of Assumption 2 imposes restrictions on the *relative magnitudes* of the organic composition in agriculture (OCC_A) and the economy-wide organic composition of capital (OCC_E). In Volume III of *Capital*, Marx worked with the assumption that the organic composition of capital in agriculture as a whole is lower than the organic composition of capital in the whole economy. The second part of Assumption 2 is a more stringent conditon because it requires that

$$z_1 \leq f(z_2) = \frac{ez_2}{(\beta - 1) z_2 + e\beta}.$$

The fact that this condition is more stringent than the simple requirement of lower OCC in agriculture than the OCC in the aggregate economy can be seen from Figure 3. The curve

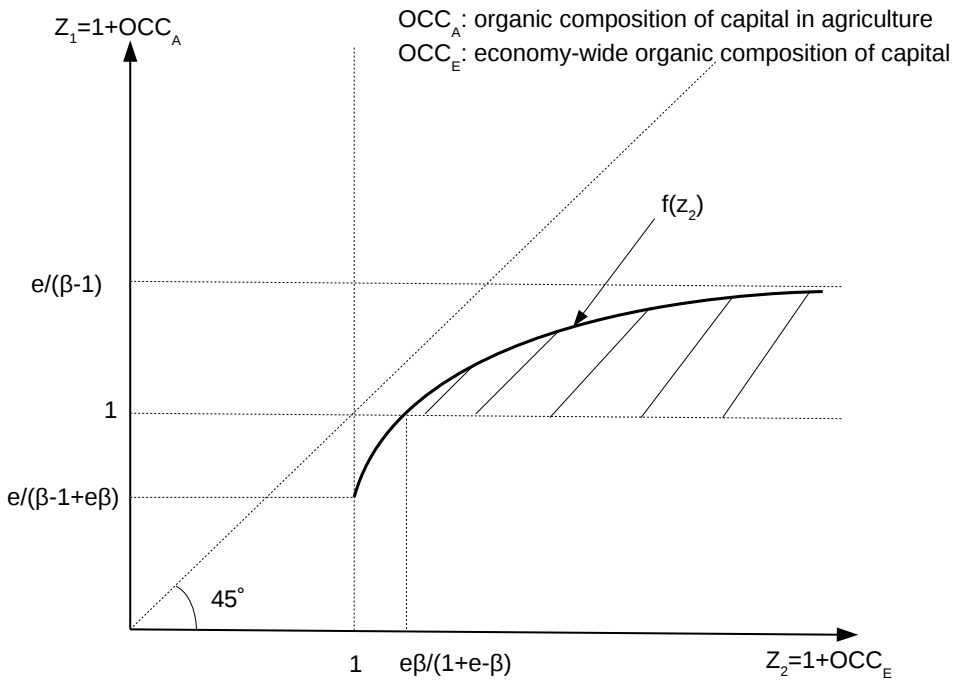

Figure 3: *Upper bound function for the organic composition in agriculture. For any value of $z_2 = 1 + OCC_E$, the function $f(z_2)$ gives the upper bound on $z_1 = 1 + OCC_A$ that ensures the result in Assumption 2.*

representing $f(z_2)$ lies completely below the 45 degree line.[14] Hence, any point on or below the curve automatically satisfies the condition that $OCC_A < OCC_E$. But there are points in Figure 3 that are above the curve and below the 45 degree line. These points satisfy the requirement that $OCC_A < OCC_E$ but do not satisfy the stronger condition required by Assumption 2.

Proposition 3. *Suppose the conditions prevailing in the economy is captured by Assumption 2. Then, we have the following:*

1. *$p^* > \tilde{p}$, i.e. the price determined by the principle of zero net flow of surplus value out of agriculture, p^*, is larger in magnitude than the price that is necessary to ensure surplus profit on the worst plot of land, $\tilde{p}$.*

2. *The total rent appropriated by the landowner of the i-th plot of land, GR_i, is positive.*

[14] This is because $f(1) < 1$ and $f'(1) < 1$.

This is because $r_i - \alpha > 0$, where r_i is the rate of profit on plot i and α is the economy-wide average rate of profit.

Proposition 3 shows that if the organic composition of capital in agriculture is 'sufficiently lower' than the economy-wide organic composition of capital and if barriers to entry ensures zero net flow of surplus value out of agriculture, then each plot of land can generate surplus profit. Since rent is merely a transformation of surplus profit, Proposition 3 demonstrates that if the organic composition of capital in agriculture is 'sufficiently lower' than the economy-wide organic composition of capital and if barriers to entry ensures zero net flow of surplus value out of agriculture then each plot of land will generate rent for its owner. Once we know the magnitude of the total rent, we can quantify the magnitudes of its three components.

There are at least three problems with the closure outlined in this section. First, it relies on a sufficiently lower organic composition of capital in agriculture compared to the rest of the economy. This might have been a reasonable assumption for the agricultural sector in 19-th century Europe, but it is a questionable assumption in much of the contemporary world (other than possibly the agricultural sector in some developing countries). Moreover, one impetus for developing a general theory of rent is to be able to apply it beyond agriculture - and in other sectors as mining, oil and natural gas, real estate, etc. In these sectors, the assumption of a lower than average organic composition of capital is even less convincing.

Second, the closure outlined in this section relies on barriers to the entry of capital into the agricultural sector due to the presence of landed property. While it is true that landed property acts as a barrier to entry of capital, that barrier is not insuperable. In fact, capital can enter into sectors that use land in capitalist commodity production. Hence, dispensing with this assumption is useful for extending the Marxist theory of rent.

Third, and possibly the most important shortcoming of the closure outlined in this section is that it does not allow the demand for the agricultural commodity to have any role in the determination of its level of output or price. In the debate between Ricardo and Malthus regarding the determination of rent, the issue of demand had been raised by the latter. Ricardo brushed aside the issue of demand arguing that it could not play any role in the determination of 'natural prices', the latter being understood as long run prices (Kurz, 1978, pp. 16–17). Marx seemed to have followed Ricardo in this respect, and later neo-Ricardian analysts have also evaded the issue of demand by taking the level of agricultural output as given (Kurz, 1978). But it seems a theoretically better strategy to allow for the role of demand in the determination of output and price.

5 Determination of the Price Level: Second Closure

Recall that we are studying an agricultural economy, in which there are N plots of land indexed by $i = 1, 2, 3, \ldots, N$, with each plot being owned by one capitalist-farmer. If N is large, each capitalist-farmer takes the price of the agricultural commodity as given and decides the level of output to maximize her profit.

If y_i and $t_i(y_i)$ denote the level of output and the total cost of production, respectively, on plot i, then profit income is given by

$$\pi_i(y_i; p) = py_i - t_i(y_i)$$

where p denotes the exogenously given (for the individual capitalist farmer) price level and the total cost of production is a function of the level of output.[15]

A plausible behavioral assumption is that capitalist farmers choose the level of output to maximize their profit income. Hence, the profit-maximizing (or optimal) level of output of the agricultural commodity on the i-th plot will be determined by the following condition:

$$p = m_i(y_i), \quad i = 1, 2, 3, \ldots, N, \tag{10}$$

where

$$m_i(y_i) = t_i'(y_i)$$

is the marginal cost of production. The conditions in (10) tell us that on each of the plots, the optimal level of output is determined by equating the marginal cost of production with the exogenously given price level (because of lack of monopoly power of any individual capitalist-farmer).

Since the amount of land is fixed, it is plausible to assume that each capitalist-farmer is operating with an increasing marginal cost of production technology. One way to capture this is to assume that $m_i(y_i)$ is a monotonically increasing function of the level of output, y_i. Hence, we can always find an inverse (function) of the marginal cost of production function to get the profit-maximizing level of output as

$$y_i(p) = m_i^{-1}(p), \tag{11}$$

[15] The fact that I allow the total cost of production to vary with the level of output means that I depart from the use of linear technologies that are used in neo-Ricardian analyses.

which gives us the supply function for each capitalist-farmer, i.e. it determines the level of output that a profit-maximizing farmer will choose to produce as a function of the price level. Since the marginal cost of production is monotonically increasing, each of the individual supply functions, being the inverse of the marginal cost of production function, will be monotonically increasing as well.

The total supply function for the agricultural output is the sum of the individual supply functions. Denoting the total supply function for the agricultural output as $S(p)$, we will have

$$S(p) = \sum_{i=1}^{N} y_i(p) = \sum_{i=1}^{N} m_i^{-1}(p). \tag{12}$$

with $S' > 0$ because each of the individual supply functions have $y_i'(p) > 0$. If $D(p; z)$ denotes the total demand function for the agricultural commodity, with z denoting non-price shift factors and $\partial D/\partial p < 0$, the equilibrium level of the price will be such that demand and supply are equal

$$S(p) = D(p). \tag{13}$$

The co-determination of the optimal level of output on the i-th plot of land and the price level of the agricultural commodity is depicted in Figure 4. On the left panel, we see the determination of the output level on the i-th plot, and on the right panel, we see the determination of the price level. On the left panel, quantity of output is measured on the horizontal axis, and price is measured on the vertical axis. MC and the profit-adjusted AVC denote the marginal and the profit-adjusted average variable cost of production curves, respectively.[16] If the price level were denoted by OB (or EC), then the profit-maximizing level of output would be given by OE, the point where the MC curve is equal to the price level.

The determination of the equilibrium level of the price in the market for the agricultural commodity is depicted in the right panel of Figure 4. The upward sloping supply curve, S, is the sum of the individual supply curves given in (12). The downward sloping demand curve,

[16]The profit-adjusted AVC curve plots $(1 + \alpha) * AVC$, where α is the economy-wide average rate of profit and AVC denotes the average variable cost of production. Given the relationship between averages and marginals, the MC curve intersects the AVC at the lowest point of the latter. The minimum point of the AVC is also the minimum point of the profit-adjusted AVC. Since the MC curve is upward-sloping, it intersects the profit-adjusted curve to the right of the minimum point, as shown in Figure 4.

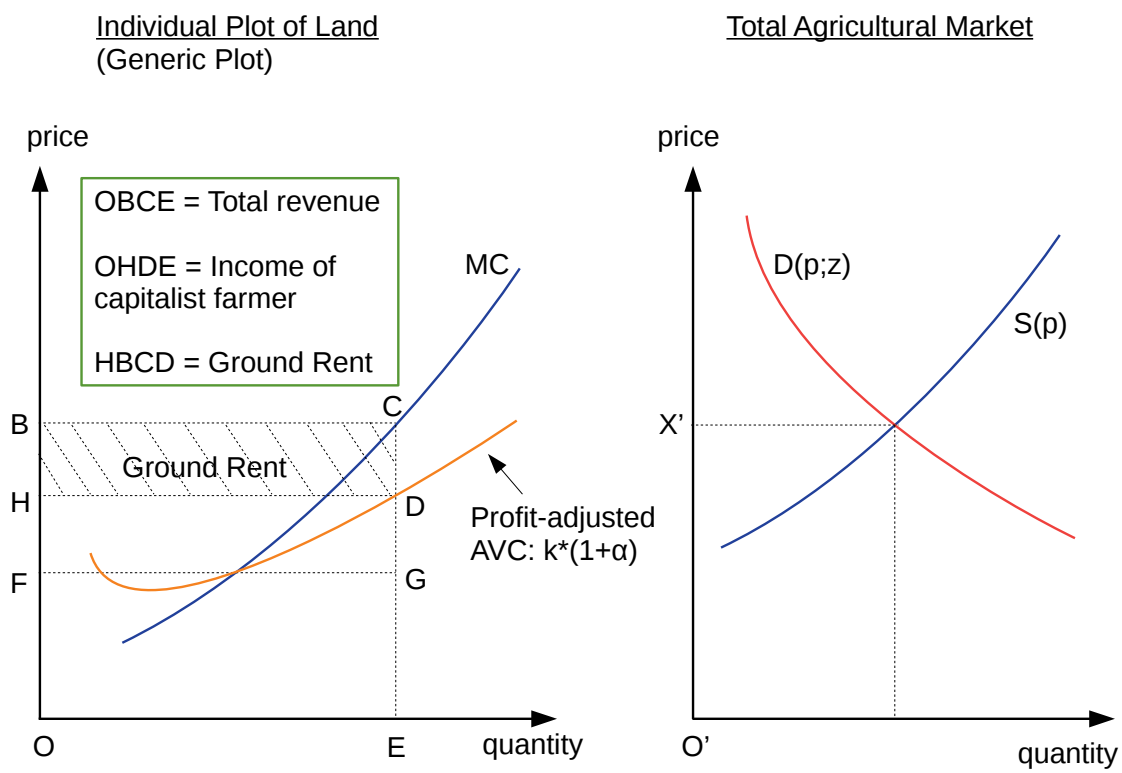

Figure 4: *Determination of equilibrium level of price and quantity in the market for the agricultural commodity. The profit-adjusted AVC curve gives* $(1+\alpha)(c+v)$.

D, is given by exogenous factors, z. The intersection of the two curves gives the equilibrium level of the price, which is represented by the height O'B' (= OB).

5.1 Determination of Rent

Once the price and output levels have been determined, we can determine the levels of rent by re-working the analysis presented in the previous sections of this paper. Let GR_i denote the rent on plot i. We know that the magnitude of rent is the surplus profit. Hence,

$$GR_i = py_i - (1+\alpha)\, y_i k_i(y_i) \tag{14}$$

where $k_i(y_i) = \{c_i(y_i) + v_i(y_i)\}/y_i$ is the average variable cost of production (AVC), and α is the economy-wide average rate of profit.[17] This is the same expression as given in (5). To see this, let r_i denote the *pre-rent* rate of profit realized on the i-th plot. Hence,

$$r_i = \frac{py_i}{c_i + v_i} - 1 \tag{15}$$

where c_i and v_i denote the constant and variable capital advanced on the i-th plot of land. Then, total rent on the i-th plot is given by

$$GR_i = (c_i + v_i)(r_i - \alpha),$$

which is the same expression as the one given in (5).

We can use Figure 4 to pin down the magnitude of rent. For the situation depicted in Figure 4, the price level is given by OB and the optimal level of output on the individual plot is given by OE. The rectangle OBCE represents the total revenue, the rectangle OHDE represents the amount of income of the capitalist farmer, and the rectangle BCDH - the shaded area - represents the rent income of the landlord. Once we know the total rent, we can break it up into its three components: absolute rent, differential rent of the first variety (DRI), and differential rent of the second variety (DRII): where

$$DRI_i = (c_i + v_i)(r_i - r_1)$$

is differential rent of the first variety,

$$DRII_i = [(c_i + v_i) - (c_1 + v_1)]\,(r_1 - \alpha)$$

is differential rent of the second variety, and

$$AR = (c_1 + v_1)(r_1 - \alpha)$$

is absolute rent.

[17] In Figure 4, we plot the profit-adjusted AVC, which is $(1 + \alpha)\,k_i(y_i)$.

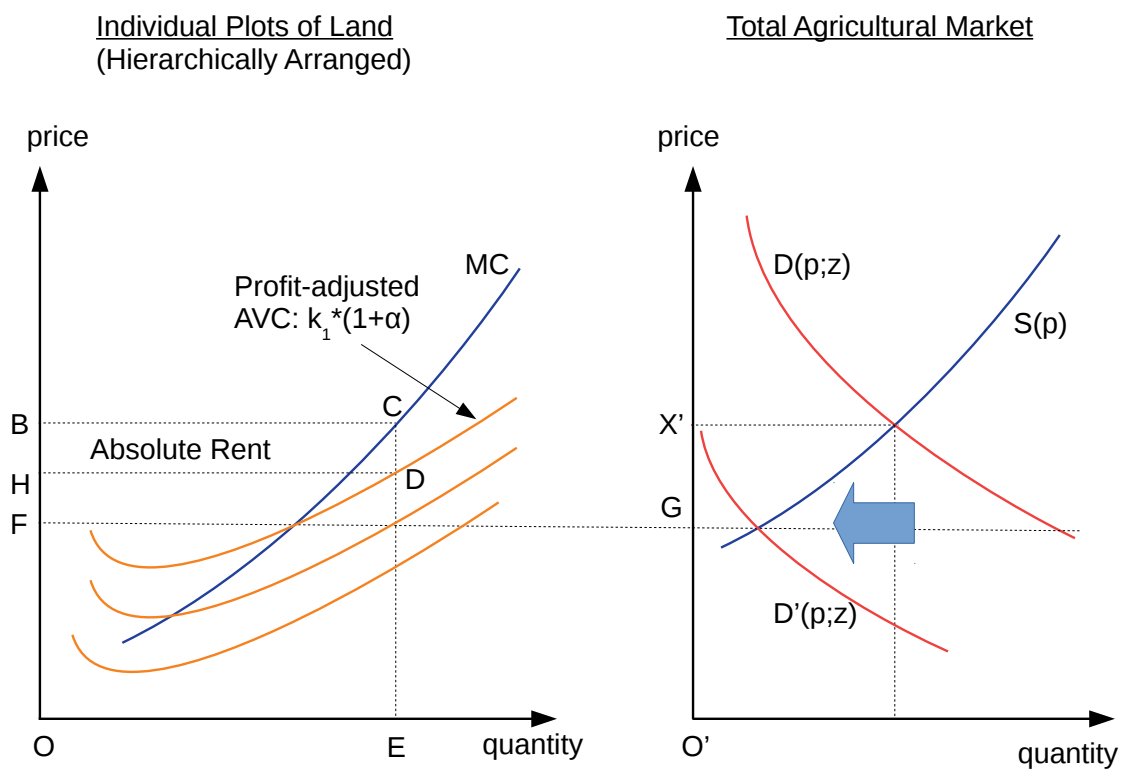

Figure 5: *Total rent, absolute and differential rent on every plot of land. The figure also shows the lower bound of demand that is necessary to ensure that all plots of land are used.*

5.2 Demand and Profit Maximization

We can use Figure 5 to get some intuition about the conditions for the generation of rent and the role of demand (for the agricultural commodity) in that process. In left panel of Figure 5, we show the situation in multiple plots of land. The profit-adjusted AVC for the worst plot is the highest. As we move from the worst to better quality plots of land, the profit-adjusted AVC shift down, as shown in the left panel of Figure 5. With multiple plots of land, we can also identify the absolute and differential rent. If the demand curve is $D(p; z)$ and the supply curve is $S(p)$, then the equilibrium price is given by $O'X'$ in the right panel of Figure 5. We can use this in the left panel to see that absolute rent is represented by the rectangle $BCDH$ (the total rent on the worst plot); and the differential rent, on any plot,

is the difference between the total rent and the absolute rent.

Under capitalist relations of production, a plot of land will be used for cultivation only when two conditions are satisfied: (a) it generates a positive magnitude of rent for the landlord, and (b) the capital invested on it by the capitalist farmer gives her a rate of profit of α (the economy-wide average rate of profit). In terms of Figure 5, this puts a lower bound on the price level: if the equilibrium price level falls below OF, then no capitalist farmer will invest her capital on the worst plot of land (because the revenue will not cover the variable cost of production). This, in turn, puts a lower bound on the level of demand (depicted as $D'(p; z)$): if the level of demand for the agricultural commodity is such that the equilibrium price level fall below OF, then the worst plot of land will lie unused.

An important point about the analysis presented in this section is that by explicitly incorporating profit-maximizing behavior, I address the possible problem that the rent calculations are sustainable. Once rents are computed on the basis of output levels that satisfy the marginal cost conditions given in (10), then there will be no incentive for capitalist farmers to deviate from those output levels. This will imply that there will be no possibility for surplus profit to remain unallocated to rent. Thus, the computation of rent (by landlords) will be mutually consistent with profit-maximizing behavior (by capitalist farmers).

5.3 Some Comparative Statics

There are two interesting comparative static exercises that could be easily carried out with the help of Figure 6. First, if there is an exogenous increase in demand, other things remaining constant, there will be an increase in the magnitude of rent. To see this, note that the exogenous increase in demand will shift the demand curve in the right panel of Figure 6 rightward. The result will be an increase in both the equilibrium price and quantity. This will increase the area of the rectangle BCDH, which shows that the magnitude of rent will increase.[18]

Second, if there is technological progress in agriculture, it will lead to an ambiguous change in the magnitude of rent. To see this, note that technological progress will lead to a downward movement of the marginal cost curves, so that the total supply curve in the

[18] In the examples Marx used in Chapter 44 of Volume III of *Capital*, an increase in the demand for the agricultural output is assumed to lead to a change in the output only on the marginal plot. Later scholars like Fine (1979) have followed Marx in this respect. But this is a rather restrictive assumption. In any realistic scenario, an increase in demand will have an impact on the production decisions on all plots of land, as can be seen in Figure 6. In the model outlined in this section, we do not need to impose the restrictive assumption.

right panel of Figure 6 will shift downward (not shown in the Figure). This will lead to an increase in the quantity of output and a fall in its price. Depending on the elasticity of the supply curve, this can lead to either a fall or a rise in the magnitude of rent. More inelastic the demand, higher is the possibility of a fall in the rent.

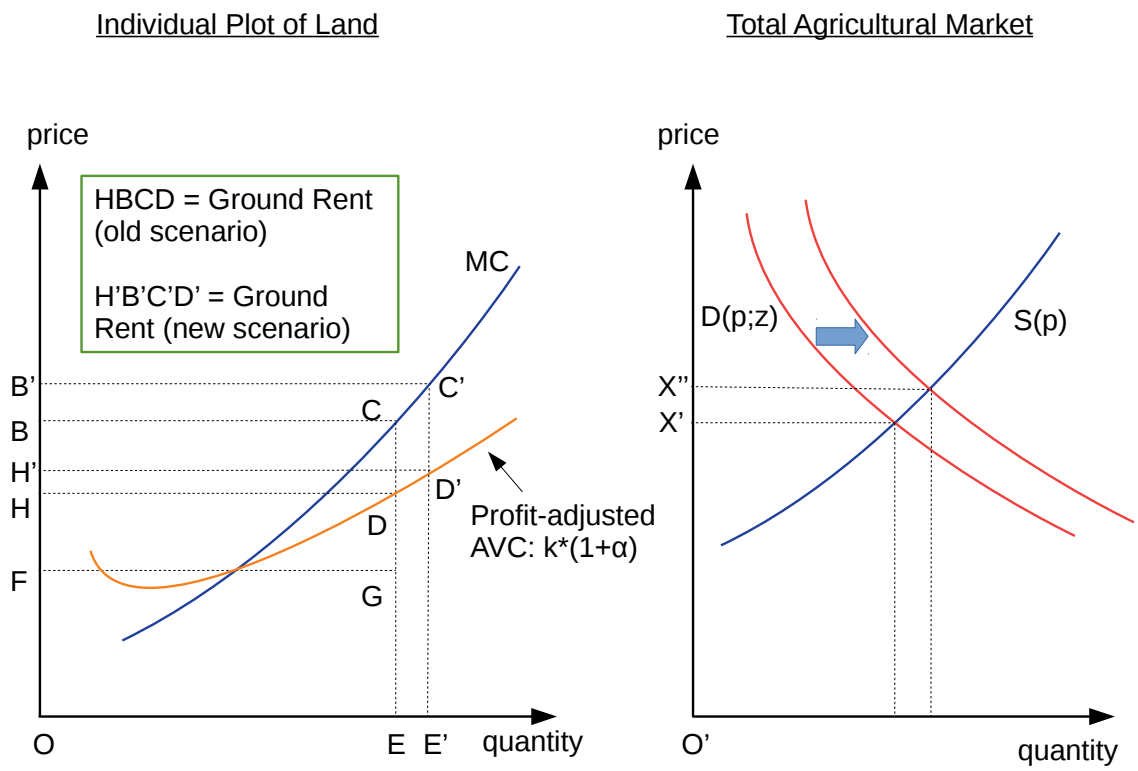

Figure 6: *Comparative statics of an exogenous increase in the demand for the commodity produced on land.*

6 Conclusion and Caveats

In this paper, I have developed a Marxist theory of rent. In developing this theory, I have used the term "land" for any nonreprdoducible natural resource that is privately owned and can be used in the production of commodities; I have used the term "agriculture" to refer

to the economic activity of producing some commodity on the "land"; and I have used the term "agricultural commodity" to refer to the commodity that is produced on the "land". An immediate implication is that the Marxist theory of rent developed in this paper is applicable far beyond agriculture proper. It can be applied to any situation where a privately owned, scarce, nonreproducible resource can be used in capitalist commodity production - for instance, in sectors like mining, oil & natural gas, real estate, and even tourism. In such situations, owners of the resource can bargain away a part of the surplus value (or national income over and above wages) generated in capitalist commodity production. This fraction is what classical economists like Ricardo and Marx understood as rent.

In this paper, I have demonstrated that the total rent can be decomposed into two components: differential rent and absolute rent. Differential rent, in turn, can be decomposed into differential rent of the first variety (which arises from differences in the quality of plots of land) and differential rent of the second variety (which arises from the differences in the magnitude of capital investment on different plots of land). Absolute rent, on the other hand, arises from the ability of the class of landowners to prevent even the worst quality of land being given *gratis* for agricultural production.

After highlighting the key role played by the price of the agricultural commodity in determining the magnitude of rent, I offered two closures of the model. In the first closure, the price of the commodity is determined by the principle of zero net flow of surplus value from agriculture, an argument pursued by Marx in Volume III of *Capital*. If the organic composition of capital is sufficiently lower than the economy-wide average, this principle of price determination can ensure positive rent on all plots of land. In the second closure, I allow for the explicit role of demand in determining both the output and price of the agricultural commodity. In the second closure, we can dispense with the requirement of lower-than-average organic composition of capital. Hence, this increases the applicability of the Marxist theory of rent to sectors beyond agriculture and to contemporary times as well.

In this paper, I have worked with a case where agricultural producers are price takers. This can be easily amended to take account of market power and price-making behaviour. If the market for the agricultural commodity is dominated by a few firms, then the market price will be higher than the competitive price that I have used in section 5. This will imply that the rent on each plot of land, including the worst plot, will be higher. Hence, in terms of the three fold decomposition of total rent that Marx discussed, price-making behavior in the market for the agricultural commodity will lead to higher absolute rent. All the qualitative features of the analysis will remain unchanged.

I would like to end the paper with two caveats. First, I have abstracted from technological change. Hence the analysis in this paper is static and only investigates the question of the distribution of a given magnitude of surplus value between capitalists and owners of natural resources. To make the analysis more realistic, one will need to incorporate technological change into the framework. Second, the analysis is partial equilibrium in orientation. It takes the average rate of profit outside agricultural production as given and investigates the issues of production and distribution within agriculture. To make the analysis more robust, it will be necessary to extend it to a general equilibrium setting where the average rate of profit and rent is jointly determined by demand and technology.

Appendix

Proof of Proposition 1

Proof. Let us denote the price of a unit of the agricultural commodity as p and note that it will be determined by the cost of production in the *worst* plot of land (Marx, 1993, pp. 797). Since the rate of profit earned by the capitalist-farmer with the worst quality of land, i.e. the capitalist-farmer indexed by $i = 1$, is given by r_1,

$$py_1 = (c_1 + v_1) + r_1 (c_1 + v_1)$$

so that the price of an unit of the agricultural commodity is given by

$$p = \left(\frac{c_1 + v_1}{y_1}\right)(1 + r_1) = k_1 (1 + r_1) . \tag{16}$$

All capitalist-farmers will be able to sell their output at the price given in (16). Hence, the revenue earned by capitalist-farmer i is given by py_i. Since the total cost of production on plot i is $(c_i + v_i)$, using the expression for the price of the agricultural commodity given in (16), we see that the total profit earned by capitalist i is given by

$$\pi_i = py_i - (c_i + v_i) = k_1 y_i (1 + r_1) - (c_i + v_i) , \quad i = 1, 2, 3, \ldots, N. \tag{17}$$

Hence, the rate of profit earned by capitalist-farmer i is given by

$$r_i = \frac{\pi_i}{c_i + v_i} = \left[\frac{k_1}{k_i}(1 + r_1) - 1\right] . \tag{18}$$

Hence,

$$\frac{1+r_{i+1}}{1+r_i} = \frac{k_i}{k_{i+1}} > 1,$$

where the last inequality comes from the use of (2). Hence,

$$r_1 < r_2 < \cdots < r_N. \tag{19}$$

This completes the proof. □

We will need the following lemma for the proof of the next proposition.

Lemma 1. *Let $\overline{p}$ denote the price of the agricultural commodity at which agriculture as a whole earns the economy-wide average rate of profit, α, and let $\tilde{p}$ denote the price of the agricultural commodity which ensures the economy-wide average rate rate of profit, α, for the worst plot of land; then $\overline{p} < \tilde{p}$.*

Proof. Since $\overline{p}$ denotes the price of the agricultural commodity at which agriculture as a whole earns the *economy-wide average rate of profit*, α, we have

$$\overline{p} = \left(\frac{C+V}{Y}\right)(1+\alpha). \tag{20}$$

Using (20) and (7), we see that

$$\frac{\tilde{p}}{\overline{p}} = \frac{(c_1+v_1)/y_1}{(C+V)/Y}.$$

We need to prove that

$$(c_1+v_1)/y_1 > (C+V)/Y.$$

Note that

$$\frac{C+V}{Y} = \frac{1}{Y}\sum_{i=1}^{N} c_i + v_i = \sum_{i=1}^{N}\left(\frac{c_i+v_i}{y_i}\right)\left(\frac{y_i}{Y}\right) = \sum_{i=1}^{N} k_i\lambda_i$$

where $k_i = (c_i+v_i)/y_i$ is the average cost of production on plot i and $\lambda_i = y_i/Y$ so that for

$i = 1, 2, \ldots, N$, $0 < \lambda_i < 1$. Under Assumption 1 and the fact that $0 < \lambda_i < 1$, we have

$$\lambda_i k_1 > \lambda_i k_i, \quad i = 2, 3, \ldots, N.$$

Hence

$$\sum_{i=2}^{N} \lambda_i k_1 > \sum_{i=2}^{N} \lambda_i k_i$$

so that adding $\lambda_1 k_1$ to both sides, we have

$$\lambda_1 k_1 + \sum_{i=2}^{N} \lambda_i k_1 > \lambda_1 k_1 + \sum_{i=2}^{N} \lambda_i k_i = \sum_{i=1}^{N} \lambda_i k_i$$

which shows that

$$k_1 \sum_{i=1}^{N} \lambda_i > \sum_{i=1}^{N} \lambda_i k_i.$$

Since $\sum_{i=1}^{N} \lambda_i = 1$, this shows that

$$k_1 > \sum_{i=1}^{N} \lambda_i k_i.$$

Since $(c_1 + v_1)/y_1 = k_1$ and $\sum_{i=1}^{N} \lambda_i k_i = (C + V)/Y$, this completes the proof. □

Proof of Proposition 3

Proof. Part 1: Note that, by Lemma 1, $\beta > 1$. Hence $(\beta - 1) z_2 + e\beta > 0$ (since $z_2, e > 0$). Thus, if

$$z_1 < \frac{e z_2}{(\beta - 1) z_2 + e\beta}$$

then, multiplying through by $(\beta - 1) z_2 + e\beta$, we have

$$\begin{aligned} z_1 z_2 (\beta - 1) + e\beta z_1 &< e z_2 \\ \beta (z_1 z_2 + e z_1) &< z_1 z_2 + e z_2 \end{aligned}$$

which, on division through by $z_1 z_2$ (which is a positive quantity), gives

$$\begin{aligned}
\beta\left(1+\frac{e}{z_2}\right) &< \left(1+\frac{e}{z_1}\right) \\
\beta\left(1+\frac{e}{1+OCC_E}\right) &< \left(1+\frac{e}{1+OCC_A}\right) \\
\left(\frac{c_1+v_1}{y_1}\right)\left(1+\frac{e}{1+OCC_E}\right) &< \left(\frac{C+V}{Y}\right)\left(1+\frac{e}{1+OCC_A}\right) \\
\tilde{p} = \left(\frac{c_1+v_1}{y_1}\right)(1+\overline{r}) &< \left(\frac{C+V}{Y}\right)(1+r^*) = p^*.
\end{aligned}$$

Part 2: By Assumption 2, $z_1 < f(z_2)$. This implies that $p^* > \tilde{p}$, i.e. the price level that arises from zero net flow of surplus value from agriculture, p^*, is larger than the price that ensures positive surplus profit on the worst plot of land, $\tilde{p}$. Hence, intuitively, when Assumption 2 holds, each plot of land generates surplus profit.

To see this more formally, let $A = p^* - \tilde{p} > 0$. Since the price of the agricultural commodity is p^*, the rate of profit on the i-th plot, r_i is given by

$$1 + r_i = \frac{p^*}{k_i} = \frac{\tilde{p}+A}{k_i} = \frac{\tilde{p}}{k_i} + \frac{A}{k_i}$$

where $k_i = (c_i + v_i)/y_i$ is the average cost of production on plot i. Hence

$$1 + r_i = \frac{\tilde{p}}{k_i} + \frac{A}{k_i} = \frac{\tilde{p}}{k_1}\frac{k_1}{k_i} + \frac{A}{k_i}\frac{k_1}{k_i}.$$

Let $\lambda_i = k_1/k_i$ for $i = 1, 2, \ldots, N$. By Assumption 1, we know that $k_1 > k_2 > \cdots > k_N$. Hence $\lambda_i > 1$ for $i = 2, \ldots, N$. Thus,

$$1 + r_i = \lambda_i \frac{\tilde{p}}{k_1} + \lambda_i \frac{A}{k_1}. \tag{21}$$

By the definition of $\tilde{p}$ as the price level that ensures the economy-wide average rate of profit on the worst plot of land, we have $\tilde{p}/k_1 = 1 + \alpha$, where α is the economy-wide average rate of profit. Hence,

$$r_i = -1 + \lambda_i (1+\alpha) + \lambda_i \frac{A}{k_1} = (\lambda_i - 1) + \lambda_i \alpha + \lambda_i \frac{A}{k_1}.$$

Hence, for $i = 1, 2, \ldots, N$,

$$r_i - \alpha = (\lambda_i - 1)(1 + \alpha) + \lambda_i \frac{A}{k_1} > 0$$

because $\lambda_1 = 1$, $\lambda_i \geq 1$, for $i = 2, \ldots, N$, $\alpha \geq 0$ and $A > 0$. This completes the proof. □

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