

# Technical Appendix to *The New York Times* Critique of Reinhart and Rogoff

*Michael Ash and Robert Pollin*

Political Economy Research Institute  
University of Massachusetts, Amherst

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This appendix provides the data and calculations underlying the material we present in our *New York Times* opinion article, “Debate and Growth: A Response to Reinhart and Rogoff” which was posted online [at this link](#) on April 29, 2013.

### CALCULATION OF MEDIAN GDP GROWTH FIGURES FOR 1946 - 2009

#### 1. Coding errors

In our original working paper with Thomas Herndon, [“Does High Public Debt Consistently Stifle Economic Growth.”](#) (HAP hereafter) we discuss in detail the coding error Reinhart and Rogoff (RR hereafter) made in their 2010 paper in the *American Economic Review*, “Growth in a Time of Debt.” This coding error occurred in their calculations of mean values over their 1946 – 2009 dataset. In fact, RR made a similar spreadsheet coding error in calculating median GDP growth by public debt/GDP category. Specifically, they calculated the median for cells in lines 30 to 44 instead of lines 30 to 49. This coding error entirely excludes five countries—Australia, Austria, Belgium, Canada, and Denmark—from their data analysis. By itself, this coding error alone reduces median GDP growth by -0.3 percentage point in RR’s highest (90 percent and above) public debt/GDP category. These same errors also lead to an overstatement in growth for lowest public debt/GDP category (0 to 30 percent) by +0.3 percentage point. Overall, this coding error, by itself, exaggerates the negative association between high public debt and GDP growth by 0.6 percentage point in the analysis of medians.

As we discuss below, the impact of this one coding error becomes magnified in combination with RR’s methodological flaws which we discuss in HAP and consider further below—that is, 1) selective exclusion of available data; and 2) a flawed weighting methodology. In Table 1 (page 2), we show the impact of this coding error on GDP growth estimates within each of the four public debt/GDP categories. Table 1 also shows the effects on growth estimates due to the selective exclusion of data and their flawed weighting methodology.

TABLE 1. CALCULATION OF MEDIAN GDP GROWTH FIGURES WITH REINHART/ROGOFF 1946 – 2009 DATASET

	Public Debt/GDP Categories			
	<i>under 30%</i>	<i>30 – 60%</i>	<i>60 – 90%</i>	<i>above 90%</i>
RR median figures in 2010 paper and <i>NY Times</i> 4/26/12	4.2%	3.0%	2.9%	1.6%
RR calculation method but with corrected spreadsheet	3.9%	3.1%	2.9%	1.9%
Recalculation with both corrected spreadsheet calculations and inclusion of Australia, Canada and New Zealand early years	3.9%	3.1%	2.9%	2.5%
Recalculation with full data and our preferred method for calculating medians	4.1%	3.1%	2.9%	2.3%

Source: Underlying data all come from RR 2010 and 2010b.

## 2. *Selective exclusion of data*

RR excluded available data for Australia (1946–1950), New Zealand (1946–1949), and Canada (1946–1950). In the Canadian case, all five omitted years were in the over 90 percent public debt/GDP category. Those years were also the only ones in which Canada is in the highest public debt/GDP category. Median GDP growth in Canada for the excluded years was 2.2 percent. For Australia as well, all five excluded years were in the highest public debt/GDP category and were the only years in which Australia was in this highest category.

The New Zealand exclusions are of particular significance. This is because all four of the excluded years were in the over 90 percent public debt/GDP category. Real GDP growth rates in those years were 7.7, 11.9, -9.9, and 10.8 percent respectively. After RR excluded these years, New Zealand contributes only one year to the highest public debt/GDP category, 1951. Real GDP growth for New Zealand in 1951 was reported as -7.6 percent. Because it is the *only* value for New Zealand that RR included in the over 90 percent public debt/GDP category, this same one year's observation at -7.6 percent is also the median value for New Zealand for the over 90 percent public debt/GDP category. If we include in the data sample the years that RR excluded, New Zealand's median GDP growth in the highest public debt/GDP category becomes +7.7 percent, as opposed to -7.6 percent. This raises serious concerns about aggregation methods that are highly sensitive to individual country's central tendencies.

As far as we know, the closest Reinhart and Rogoff have come to explicitly explaining their decision to exclude the early years of data for Australia, Canada and New Zealand is the following remark in their January 2010 NBER Working Paper (full citation in HAP):

“Of course, there is considerable variation across the countries, with some countries such as Australia and New Zealand experiencing no growth deterioration at very high debt levels. It is noteworthy, however, that those high-growth high-debt observations are clustered in the years following World War II” (p. 11).

In other words, RR appear to justify these selective exclusions because they “are clustered in the years following World War II” when economic growth was high. However, in contrast with this apparent reasoning applied to the cases of Australia, Canada and New Zealand, RR chose to include all of the immediate post World War II observations in which the United States was in the over 90 percent public debt/GDP category. In three of these years, the United States economy was contracting at the same time as it was in the highest public debt/GDP category. RR still have not provided a full explanation of their reasoning behind their decision to exclude Australia, Canada and New Zealand in these years, while these economies were growing rapidly, while including the United States while it was contracting in three of the four relevant years.

## 3. *RR's flawed data weighting methodology*

We discuss in detail in HAP why we find RR's weighting methodology to be flawed. Here we focus on the impact of applying this flawed methodology to the calculation of median GDP growth

figures for their 1946 – 2009 dataset. As we will see, RR’s flawed weighting method amplifies the effect of the exclusion of years for New Zealand so that this exclusion has a very large effect on RR’s median results.

Although RR do not document this in either of their 2010 research papers, in fact, the RR calculation of median GDP growth is a median of each country’s median GDP growth within each of their four public debt/GDP categories. More specifically, after assigning each country-year to one of the four public debt/GDP groups, RR calculates the median real GDP growth for each country within each of the four public debt/GDP categories. This has the effect of creating a single median value for the country for the years it appeared in each of the public debt/GDP categories. RR then takes the median of these country medians. This has the effect of massively amplifying the impact of early years for New Zealand, Australia, and Canada. Including the early years for these countries, median GDP growth in the over 90 percent public debt/GDP category is 2.5 percent. When RR exclude those years, as we have seen, median GDP growth in the over 90 percent public debt/GDP category is 1.9 percent. RR needs to justify their weighting methodology in detail. It otherwise appears arbitrary and unsupportable.

Our preferred approach for the median analysis is to take the median GDP growth of all the country-years appearing in each of the four public debt/GDP categories. With this approach, excluding the early years for Australia, Canada, and New Zealand finds median GDP growth in the highest public debt/GDP category to be 2.2 percent. When we then include the early years for Australia, Canada, and New Zealand, the resulting median GDP growth in the highest public debt/GDP category becomes 2.3 percent. That is, the median under our preferred method is robust to small perturbations in the data. This is not true with RR’s median of medians methodology.

In conclusion, with RR’s median of medians approach, minor adjustments of the sample, resulting, for example, from a spreadsheet error, generate a wide range of GDP growth estimates for the over 90 percent public debt/GDP category. As we summarize in Table 1, RR report 1.6 percent median GDP growth; fixing the spreadsheet error adjusts the estimate to 1.9 percent, and including available data for the early postwar increases the estimate to 2.5 percent. A more robust approach finds median growth in the over 90 percent public debt/GDP category to be between 2.2 and 2.3 percent. These results are robust regardless of whether we include the early years for Australia, Canada and New Zealand. It is also notable that these median figures are nearly identical to the average GDP growth figures for the over 90 percent public debt/GDP category that we reported in HAP.

## **RR ANALYSIS OF 1790 – 2009 DATA**

### *1. Coding errors*

As with their calculations of both averages and medians with 1946 – 2009 data, RR 2010 made a spreadsheet coding error in calculating mean GDP growth by public debt/GDP category for 20 advanced economies over the 220 year period 1790-2009. By calculating the mean for cells in lines

5 to 19 instead of lines 5 to 24, the coding error entirely excludes five countries—Australia, Austria, Belgium, Canada, and Denmark—from the analysis. This spreadsheet error alone reduces estimated mean GDP growth by 0.3 percentage point in the highest public debt/GDP category. The spreadsheet error also overstates growth in the next highest public debt/GDP category by 0.2 percentage point, erroneously expanding the reported gap between these categories by 0.5 percentage point.

## 2. *Weighting methodology*

RR’s flawed weighting method follows the following steps: 1) it sorts every country-year into public debt/GDP categories, calculates the average for each country within each category, and then averages the country averages within each category. This approach creates the possibility that a single country-year can have a disproportionate effect on the results. For example, Norway spent only one year (1946) in the 60-90 percent public debt/GDP category over the total 130 years (1880-2009) that Norway appears in the data. Norway’s economic growth in this one year was 10.2 percent. This one extraordinary growth experience contributes fully 5.3 percent (1/19) of the weight for the mean GDP growth in this category even though it constitutes only 0.2 percent (1/445) of the country-years in this category. Indeed Norway’s one year in the 60-90 percent GDP category receives equal weight to, for example, Canada’s 23 years in the category, Austria’s 35, Italy’s 39, and Spain’s 47.

Using their weighting method, RR find that GDP growth declines from 3.4 percent in the 60-90 percent public debt/GDP category to 1.7 percent in the over 90 percent category, a steep drop-off of 1.7 percentage points. In contrast, country-year weighting of the means finds GDP growth of 2.5 percent in the 60-90 percent public debt/GDP category and 2.1 percent in the over 90 percent public debt/GDP category. This is a modest difference of 0.4 percentage point. We summarize these various GDP growth rate figures over the 1790 – 2009 dataset in Table 2.

TABLE 2. CALCULATION OF AVERAGE GDP GROWTH FIGURES WITH REINHART/ROGOFF 1790 – 2009 DATASET

	Public Debt/GDP Categories			
	<i>under 30%</i>	<i>30 – 60%</i>	<i>60 – 90%</i>	<i>above 90%</i>
RR mean figures in 2010 paper and <i>NY Times</i> 4/26/12	3.7	3.0	3.4	1.7
Estimate with corrections for coding errors, selected exclusions, and RR average method	3.7	3.2	2.5	2.1

Source: Underlying data all come from RR 2010, and 2010b.

The pattern with the corrected mean figures within each public debt/GDP category casts doubt on the identification of a nonlinear response which was an important component of RR’s findings. We explore this issue further by adding an additional public debt/GDP category. Specifically, we add 90–120 percent and greater-than-120 percent public debt/GDP categories. Average GDP growth is computed as the mean over country-years in each category. We see the results in Table 3. As we see there, far from appearing to be a break, average real GDP growth in the

category of public debt/GDP between 90 and 120 percent is 2.5 percent—identical to average GDP growth in the 60–90 percent category. Even with the new highest grouping of over 120 percent public debt/GDP, average GDP growth is lower at 1.6 percent but does not fall in a sharp non-linear pattern. Thus, with RR’s roughly 220-year dataset, there appears to be no decline in GDP growth at the 90 percent public debt/GDP ratio, despite the fact that RR had identified this 90 percent threshold as an historic boundary.

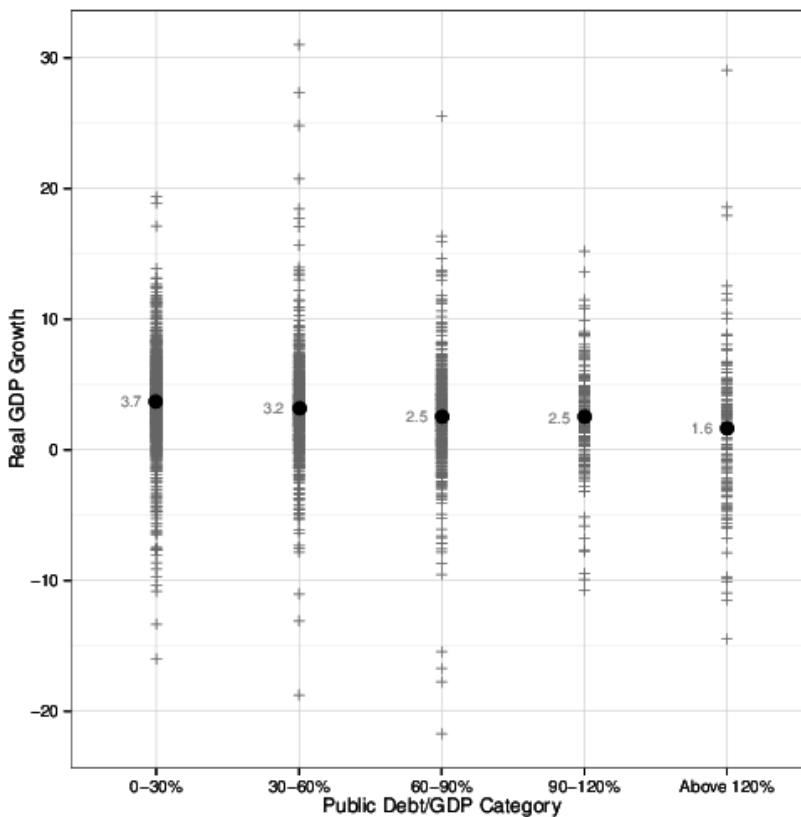
TABLE 3. RECALCULATION OF AVERAGE GDP GROWTH FIGURES WITH REINHART/ROGOFF 1790 – 2009 DATASET AND ADDITIONAL PUBLIC DEBT/GDP CATEGORY

	Public Debt/GDP Categories				
	<i>under 30%</i>	<i>30 – 60%</i>	<i>60 – 90%</i>	<i>90 – 120%</i>	<i>above 120%</i>
Estimate with corrections for coding errors, selected exclusions, and RR average method	3.7%	3.2%	2.5%	2.5%	1.6%

Source: Underlying data all come from RR 2010 and 2010b.

In Figure 1, we present all of the country-year data for RR’s 220-year dataset. The figure also shows the similarity of the means across public debt/GDP category and the large variation in real GDP growth within each public debt/GDP category.

FIGURE 1. REAL GDP GROWTH BY EXPANDED PUBLIC DEBT/GDP CATEGORIES, COUNTRY-YEARS, 1790–2009



Notes. The unit of observation in the scatter diagram is country-year with real GDP growth plotted against five debt/GDP categories. Average real GDP growth for all country-years within category are printed to the left.

Source: Authors’ calculations from working spreadsheet provided by RR.

## THE RELATIONSHIP BETWEEN HIGH PUBLIC DEBT AND ECONOMIC GROWTH IN RECENT YEARS

In HAP, we explored the historical specificity of the association between public debt and GDP growth by examining average real GDP growth by public debt category for subsampled periods of the data. Here we focus on the relationship during the final decade of the analysis, 2000-2009, a period that includes the Great Recession.

Only four countries appear in the highest public debt/GDP category in this period. They collectively contribute 31 country-years: Belgium (8 years), Greece (10 years), Italy (3 years), and Japan (10 years). Mean GDP growth by public debt/GDP category is shown in Table 4. As we can see, over this 2000 – 2009 period, real GDP growth in the over 90 percent public debt/GDP category has actually outperformed GDP growth in the 60 – 90 percent public debt/GDP category.

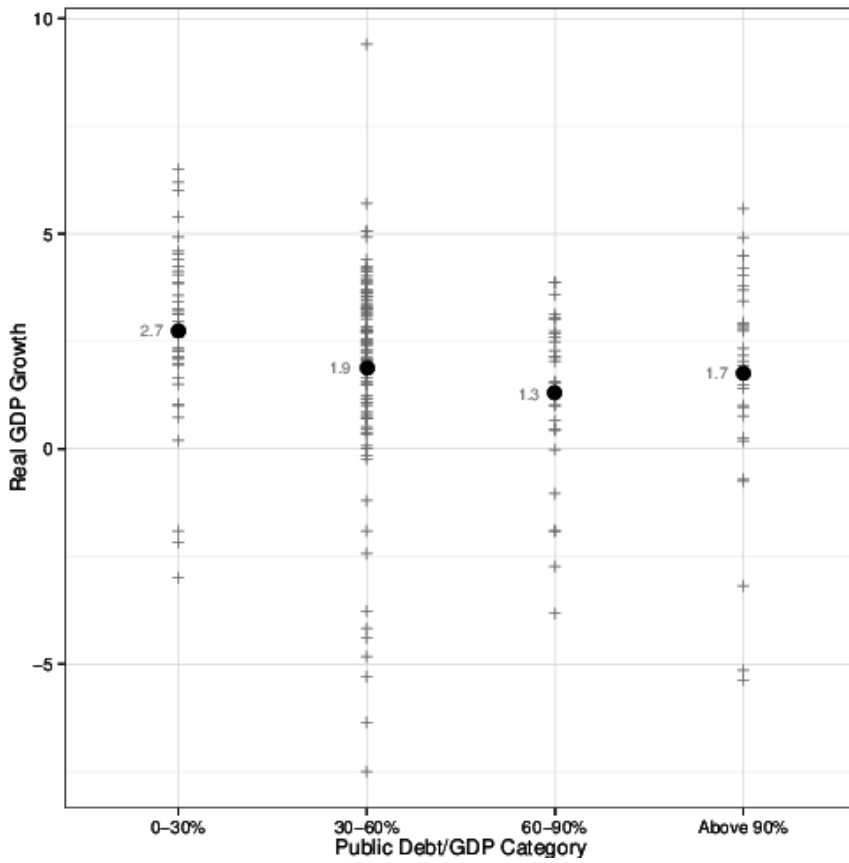
TABLE 4. CALCULATION OF AVERAGE GDP GROWTH FIGURES WITH 2000 – 2009 DATA ONLY (STANDARD ERRORS IN PARENTHESES)

	Public Debt/GDP Categories			
	<i>Under 30%</i>	<i>30 – 60%</i>	<i>60 – 90%</i>	<i>above 90%</i>
Estimate with corrections for coding errors, selected exclusions, and RR average method	2.7% (0.3)	1.9% (0.3)	1.3% (0.4)	1.7% (0.5)

Source: Underlying data all come from RR 2010 and 2010b.

As we mention above, the number of observations is relatively small. Standard errors are consequently high. We can see the detailed pattern more fully in Figure 2 (page 7), which presents all of the country-year data for all countries in the 2000-2009 data. As we see, there is certainly no sharp drop off in economic growth once the 90 percent threshold is dropped. Rather, the relationship between public debt and GDP growth is even weaker in more recent years than in the earlier years of the sample.

FIGURE 2: REAL GDP GROWTH BY PUBLIC DEBT/GDP CATEGORIES,  
COUNTRY-YEARS, 2000-2009 (IN PERCENTAGES)



Notes. The unit of observation in the scatter diagram is country-year with real GDP growth plotted against four public debt/GDP categories.

Source: Authors' calculations from working spreadsheet provided by RR.